Math 361: Real Analysis 2
Assignment # 14

Remember you may use anything we proved in class or previous homework assignments, previous problems in the same homework assignment and even previous parts from the same question even if you did not complete them. The one exception is that if I am asking you to work on part of a proof of a theorem from class you may not use that theorem in your proof (but you may use any part we proved before the part you are proving).

You may also use the major parts of Real I. If you have a question about that please ask.

1. (a) Find $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(n+3)n!} = \lim_{N\to\infty} \sum_{n=0}^{N} \frac{1}{(n+3)n!}$. (Hint: Modify an exam problem.)

   (b) Show $e > \frac{122}{45}$.

2. (a) Let $h \in c^1(\mathbb{R})$ be such that $h' = h$. Show $h(x) = h(0)\exp(x)$.

   (b) Use this fact (and not how we did it before with properties of ln) that $\exp(a+b) = \exp(a)\exp(b)$.

3. Let $0 < L < 1$ and $f_n(x) = \displaystyle\sum_{k=0}^{n} x^k$ and $f(x) = \dfrac{1}{1-x}$ defined on $(-1, 1)$.

   (a) Show that f_n converges to f pointwise on $(-1, 1)$.

   (b) Show that f_n converges uniformly to f on $[-L, L]$.

   (c) Find:

   $$\lim_{n\to\infty} \int_{-\frac{1}{3}}^{\frac{1}{2}} f_n.$$

4. Let $A \subseteq \mathbb{R}$. Suppose that $f, f_n g, g_n \in b(A)$ for all $n \in \mathbb{N}$. Do the following:

   (a) On the exam you showed that on A, $||fg||_u \leq ||f||_u ||g||_u$. Give a concrete example to show that it need not be the case that $||fg||_u = ||f||_u ||g||_u$.

   (b) Show that if $f_n \to_u f$ then $||f_n||_u \to ||f||_u$

   (c) Show that if $f_n \to_u f$ and $g_n \to_u g$ on A then $f_n g_n \to_u fg$ on A.