# Math 361: Real Analysis 2
# Assignment # 15

Remember you may use anything we proved in class or previous homework assignments, previous problems in the same homework assignment and even previous parts from the same question even if you did not complete them. The one exception is that if I am asking you to work on part of a proof of a theorem from class you may not use that theorem in your proof (but you may use any part we proved before the part you are proving).

You may also use the major parts of Real I. If you have a question about that please ask.

1. Suppose that function $\{S_n\}$ and $\{C_n\}$ are a sequence of functions, such that $c_0 = 1$, for each $n \in \mathbb{N}$, $S_n(x) = \int_0^x C_n$ and $C_{n+1}(x) = 1 - \int_0^x S_n$. Prove by induction that for all $n \in \mathbb{N}$:
   (a) $S_n(x) = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$
   (b) $C_n(x) = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$

   Hint: Just do one induction to do both of these.

2. Let $0 < L < 1$ and $f_n(x) = \sum_{k=0}^{n} x^k$ and $f(x) = \frac{1}{1-x}$ defined on $(-1,1)$. Remember you showed that $f_n$ converges uniformly to $f$ on $[-L, L]$.
   (a) Find $g$ defined on $(-1,1)]$ so that that $\sum_{k=0}^{n}(k+1)x^k \to_u g$ on $[-L, L]$.
   (b) Find $\sum_{n=0}^{\infty} \frac{n+1}{2^n}$.

3. For $x, y \in \mathbb{R}$ find $C(x+y)$ and $C(-x)$ in terms of $S(x)$ and $C(x)$.

4. Suppose $\cosh(x), \sinh(x)$ are functions defined on $\mathbb{R}$ such that $\cosh'(x) = \sinh(x)$ and $\sinh'(x) = \cosh(x)$ with $\cosh(0) = 1$ and $\sinh(0) = 0$.
   (a) Show that $\cosh^2(x) - \sinh^2(x) = 1$ for all $x \in \mathbb{R}$.
   (b) Show that if $f$ is twice differentiable on $\mathbb{R}$ with $f''(x) = f(x)$ then there exists $\alpha$ and $\beta$ such that $f(x) = \alpha \cosh(x) + \beta \sinh(x)$. Also find $\alpha$ and $\beta$ in terms of $f$.
   (c) Find $\sinh(x+y)$ and $\cosh(x+y)$ for $x, y \in \mathbb{R}$ in terms of $\sinh(x)$ and $\cosh(x)$.
   (d) Determine if sinh and cosh are odd, even or neither.