Math 361: Real Analysis 2
Assignment # 16

Remember you may use anything we proved in class or previous homework assignments, previous problems in the same homework assignment and even previous parts from the same question even if you did not complete them. The one exception is that if I am asking you to work on part of a proof of a theorem from class you may not use that theorem in your proof (but you may use any part we proved before the part you are proving).

You may also use the major parts of Real I. If you have a question about that please ask.

Remember what we called $S(x)$ and $C(x)$ we are now calling $\sin(x)$ and $\cos(x)$ respectively.

1. (a) Show that the only roots of the cos function on $[0, 2\pi]$ are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

   (b) Show that cos is periodic with period 2π.

2. Show that $3 \leq \pi \leq 2\sqrt{6 - 2\sqrt{3}}$. (Hint use $\cos(x)$)

3. (a) Show that sin has roots when $x = k\pi$ where $k \in \mathbb{Z}$.

   (b) Show these are the only roots of sin.

4. Remember in a previous homework we defined:

$$I_n = \frac{q^{2n}}{n!} \int_0^a x^n (a-x)^n g(x)\, dx$$

   Where $q > 0$, $a > 0$ and $g \in c([0, a])$ with $g(0) = g(a) = 0$ and $0 < g(x) \leq M$ on $(0, a)$. and $g''(x) = -g(x)$ on $[0, a]$.

   (a) Show that $g(x) = \sin(x)$ and $a = \pi$, satisfies these conditions. For now we will define I_n with these choices.

   (b) Find I_0.

   (c) Find I_1.

   (d) Show that if $q \in \mathbb{N}$ then $I_0, I_1 \in \mathbb{N}$.

   (e) Write the recursion from a previous homework with these choices.