Math 361: Real Analysis 2
Assignment # 17

Remember you may use anything we proved in class or previous homework assignments, previous problems in the same homework assignment and even previous parts from the same question even if you did not complete them. The one exception is that if I am asking you to work on part of a proof of a theorem from class you may not use that theorem in your proof (but you may use any part we proved before the part you are proving).

You may also use the major parts of Real I. If you have a question about that please ask.

Remember what we called $S(x)$ and $C(x)$ we are now calling $\sin(x)$ and $\cos(x)$ respectively.

1. Define $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$.
   (a) Show $\tan(x) : (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$ is an increasing, continuous, bijection.
   (b) Define $\arctan(x) : \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2})$ to be the functional inverse of $\tan(x)$. Show it is an increasing, continuous, bijection.
   (c) Find $\arctan'(x)$.
2. Show if $\sum_{k=1}^{\infty} a_k$ converges then $\lim_{k\to\infty} a_k = 0$.
3. Suppose for some $M > 0$, $0 \leq Mb_k \leq a_k$ eventually. Show that if $\sum b_k$ diverges, then $\sum a_k$ diverges.
4. Suppose that $\left|\dfrac{a_{k+1}}{a_k}\right| > 1$ eventually. Show that $\sum a_k$ diverges.