Math 361: Real Analysis 2
Assignment # 18

Remember you may use anything we proved in class or previous homework assignments, previous problems in the same homework assignment and even previous parts from the same question even if you did not complete them. The one exception is that if I am asking you to work on part of a proof of a theorem from class you may not use that theorem in your proof (but you may use any part we proved before the part you are proving).

You may also use the major parts of Real I. If you have a question about that please ask.

1. Suppose that $\sum_{k=0}^{\infty} a_k(x_0 - a)^k$ diverges for some $x_0 \in \mathbb{R}$. Show that the power series diverges for all x such that $|x - a| > |x_0 - a|$.

2. Suppose that for a power series $\sum_{k=0}^{\infty} a_k(x_0 - a)^k$ has radius of convergence R. That is:

   $$R = \sup\{|x - a| : \text{ the series converges for } x\}$$

   which is possible ∞.

   Show:

   (a) The series converges absolutely on $(a - R, a + R)$

   (b) The series diverges for all x for which $|x - a| > R$

3. For each of the following find the radius of convergence of the series.

   (a) $\sum_{k=1}^{\infty} k(x - 2)^k$

   (b) $\sum_{k=0}^{\infty} k\left(-\frac{1}{3}\right)^k (x - 2)^k$

   (c) $\sum_{k=0}^{\infty} \frac{k}{3^{2k-1}}(x - 1)^{2k}$

   (d) $\sum_{k=2}^{\infty} \frac{x^k}{(\ln k)^k}$