Math 361: Real Analysis 2
Assignment # 19

Remember you may use anything we proved in class or previous homework assignments, previous problems in the same homework assignment and even previous parts from the same question even if you did not complete them. The one exception is that if I am asking you to work on part of a proof of a theorem from class you may not use that theorem in your proof (but you may use any part we proved before the part you are proving).

You may also use the major parts of Real I. If you have a question about that please ask.

1. Let a_n defined as:

$$a_n = \begin{cases} \frac{1}{2^{(n+1)/2}.3^{(n-1)/2}} & \text{if } n \text{ is odd} \\ \frac{1}{2^{n/2}.3^{n/2}} & \text{if } n \text{ is even} \end{cases}$$

   Show that $\sum_{k=0}^{\infty} a_k$ converges.

2. Find all x (except possibly 2 points) such that the following series converge:

   (a) $\sum_{k=0}^{\infty} \frac{(x+2)^k}{5^k}$

   (b) $\sum_{k=1}^{\infty} \frac{(x-1)^k}{k^2 3^k}$

   (c) $\sum_{k=0}^{\infty} \frac{(2x)^k}{k!}$

   (d) $\sum_{k=1}^{\infty} \left(1+\frac{x}{k}\right)^k$

   (That is you can say "I don't know" for at most 2 points per problem, but you must say what all the other x do).

3. Consider the power series: $\sum_{k=1}^{\infty} kx^k$.

   (a) Find its radius of convergence

   (b) In the interior of the interval of convergence, give a closed form expression (one without summation) for what the function the series converges to.

   (c) Find the sum of $\sum_{k=1}^{\infty} \frac{k}{2^k}$.

4. Consider the power series: $\sum_{k=1}^{\infty} k^2x^k$.

   (a) Find its radius of convergence

   (b) In the interior of the interval of convergence, give a closed form expression (one without summation) for what the function the series converges to.

   (c) Find the sum of $\sum_{k=1}^{\infty} \frac{k^2}{3^k}$.