Math 361: Real Analysis 2
Assignment # 2

1. State the Bolzano-Weierstrass Theorem

2. Use the Bolzano-Weierstrass Theorem to show that if $f : K \to \mathbb{R}$ is a continuous function where $K \subseteq \mathbb{R}$ is compact then $f(K)$ is compact. (Hint: Remember that $f : D \to \mathbb{R}$ is continuous at $c \in D$ if and only if for every sequence $\{c_n\} \subseteq D$ that converges to c, $\{f(c_n)\}$ converges to $f(c)$.

3. Show if $a, b \in \mathbb{R}$ with $a < b$ and $f : [a, b] :\to \mathbb{R}$ continuous then there exists $m, M \in \mathbb{R}$ with $m \leq M$ (note m could equal M so the "interval" would contain one element) such that $f([a, b]) = [m, M]$.

4. Consider the interval $[0, 6]$ and the function $f(x) = x^3$. For each of the following tagged partitions find:

   i $\left|\left|\dot{\mathcal{P}}\right|\right|$

   ii $\mathrm{S}(f, \dot{\mathcal{P}})$

   (a) $\dot{\mathcal{P}} = ((0, 2, 6)), (0, 5))$
   (b) $\dot{\mathcal{P}} = ((0, 1, 2, 5, 6)), (0.5, 1.5, 3.5, 5.5))$
   (c) $\dot{\mathcal{P}} = ((0, 1, 2, 3, 6)), (1, 1, 3, 3))$