Math 361: Real Analysis 2
Assignment # 23

Remember you may use anything we proved in class or previous homework assignments, previous problems in the same homework assignment and even previous parts from the same question even if you did not complete them. The one exception is that if I am asking you to work on part of a proof of a theorem from class you may not use that theorem in your proof (but you may use any part we proved before the part you are proving).

You may also use the major parts of Real I. If you have a question about that please ask.

1. Consider the Banach space $l^\infty(\mathbb{R})$. Determine if the following are bounded linear operations and if so determine the operator norm:

    1. $T_1((a_1, a_2, \ldots,)) = (1, a_1, a_2, \ldots)$
    2. $T_2((a_1, a_2, \ldots,)) = (0, a_1, 0, \frac{1}{2}a_2, 0, \frac{1}{3}a_3, 0, \frac{1}{4}a_4, \ldots)$

2. Show $\frac{\sqrt{2}}{\sqrt{\pi}}\sin(x)$ and $\frac{\sqrt{2}}{\sqrt{\pi}}\cos(x)$ form an orthonormal basis for a finite dimensional subspace of $L^2([0,\pi])$. (You might have to look up some linear algebra.)

3. Suppose X is an inner product space and fix $y \in X$. Define a function $f : X \to \mathbb{R}$ by $f(x) = \langle x, y \rangle$. Show f is continuous (using the metric space induced by inner product on X and the usual metric space on $\mathbb{R}$.)

4. (a) Let $I : (\mathcal{R}([0,1], ||\cdot||_\infty) \to \mathbb{R}$ be defined by $I(f) = \int_0^1 f$. Determine (with proof) if I is a bounded linear functional.

    (b) Let $D : C^1([0,1], ||\cdot||_\infty) \to (C([0,1]), ||\cdot||_\infty)$ be defined by $D(f) = f'$. Determine (with proof) if D is a bounded linear functional. (Hint consider $f_n = \dfrac{\sin(nx)}{n}$).