Math 361: Real Analysis 2
Assignment # 3

1. Suppose $f \in \mathcal{R}[a,b]$ and $\dot{\mathcal{P}}_n$ be any sequence of pointed partitions such that $\lim_{n\to\infty} \left|\left|\dot{\mathcal{P}}_n\right|\right| = 0$. Show that $\lim_{n\to\infty} S(f, \dot{\mathcal{P}}_n) = \int_a^b f$.

2. Let $f(x) = x^2$. Assume that $f \in \mathcal{R}[0,1]$ (we will soon show that all continuous functions are Riemann integrable). Use the fact that, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ to show that $\int_0^1 x^2 = \frac{1}{3}$.

3. Suppose $f \in b([a,b])$ and the there exists two sequences of pointed partitions such that $\lim_{n\to\infty} \left|\left|\dot{\mathcal{P}}_n\right|\right| = 0$ and $\lim_{n\to\infty} \left|\left|\dot{\mathcal{Q}}_n\right|\right| = 0$, but $\lim_{n\to\infty} S(f, \dot{\mathcal{P}}_n) \neq \lim_{n\to\infty} S(f, \dot{\mathcal{Q}}_n)$. Show $f \notin \mathcal{R}[a,b]$.

4. Consider the function:
$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$
Prove $f \notin \mathcal{R}[0,1]$.