Math 361: Real Analysis 2
Assignment # 7

Remember you may use anything we proved in class or the homework. The one exception is that if I am asking you to work on part of a proof of a theorem from class you may not use that theorem in your proof (but you may use any part we proved before the part you are proving). You may also use the major parts of Real I. If you have a question about that please ask.

1. Suppose that $f \in \mathcal{R}[a,b]$ and $a = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = b$. Then show that $f \in \mathcal{R}[x_{i-1}, x_i]$ for all $1 \leq i \leq n$ and:

$$\int_a^b f = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} f.$$

2. Let $f : [a,b] \to \mathbb{R}$. In class we showed that if f is increasing on $[a,b]$ (i.e. if $x_1 \leq x_2$ then $f(x_1) \leq f(x_2)$) then $f \in \mathcal{R}[a,b]$. Show:
   (a) Show that if f is increasing then:

$$f(a)(b-a) \leq \int_a^b f \leq f(b)(b-a)$$

   (b) Show that if f is decreasing then $f \in \mathcal{R}[a,b]$.

3. Let $h \in b([0,2])$ defined by:

$$h(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 18 & \text{if } x = 1 \\ 2x+7 & \text{if } x > 1. \end{cases}$$

   Show that $h \in \mathcal{R}[0,2]$ and find $\displaystyle\int_0^2 h$.

4. In class we are proving if $f \in \mathcal{R}[a,b]$ and $\alpha, \beta, \gamma \in [a,b]$ then:

$$\int_\alpha^\beta f = \int_\alpha^\gamma f + \int_\gamma^\beta f.$$

   Do your part by proving your order of α, β, γ.