5.2

Robriana Johnson
section 2
def. integral

1) $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{10\cancel{0}x}{\cancel{10}\sqrt{100-x^2}}\,dx$

$= \frac{10}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{x\cdot 2}{\sqrt{100-x^2}}\,dx$

$= 5\int \frac{du}{u}$

$= 5\arcsin\frac{u}{a}$

$= 5\arcsin\frac{x}{10}\Big]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$

$= 5\left(\arcsin\left(\frac{3\pi}{20}\right) - \arcsin\left(\frac{\pi}{20}\right)\right)$

$= 5\arcsin\frac{2\pi}{20}$

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a}+C$

$u = 100-x^2$

$du = 2x\,dx$

$a = 10$

2) $\int_1^6 \frac{1}{e}\,dx$

$= \frac{1}{e}x\Big]_1^6$

$= \frac{6}{e} - \frac{1}{e}$

$= \frac{5}{e}$

Karen Clark
5.3

① $\int_1^2 \frac{v^3+3v^6}{v^4} dv$

$= \int_1^2 \frac{1}{v} + 3v^2$

$= \ln v + \frac{3v^3}{3} \Big]_1^2 = \ln v + v^3 \Big]_1^2$

$= (\ln 2 + 8) - (0+1) = \boxed{\ln 2 + 7}$

② $\int_0^{\pi/3} \frac{\sin\theta + \sin\theta \tan^2\theta}{\sec^2\theta} d\theta$

$= \int_0^{\pi/3} \frac{\sin\theta(1+\tan^2\theta)}{\sec^2\theta}$

$\sec^2\theta = \tan^2\theta + 1$

$= \int_0^{\pi/3} \frac{\sin\theta(\sec^2\theta)}{\sec^2\theta}$

$= \int_0^{\pi/3} \sin\theta$

$= -\cos\theta \Big]_0^{\pi/3} = -\cos\frac{\pi}{3} + \cos 0 = \frac{-1}{2} + 1 = \boxed{\frac{1}{2}}$

Catrance Yindra

$\boxed{\begin{array}{c}5.4\\ \text{Answers}\end{array}}$

7.) $g(x) = \frac{1}{x^3+1}$

9.) $g(y) = y^2 \ln y$

5.5

Tim B
Sec 2
A

1. $\int \frac{dx}{5-3x} \rightarrow \boxed{\frac{-\ln(5-3x)}{3}}$

2. $\int_0^1 x^2(1+2x^3)^5\,dx \rightarrow \left.\frac{(2x^3+1)^6}{36}\right|_0^1 = \frac{1}{13} - \frac{1}{36}$

$\rightarrow \frac{2}{36} = \boxed{\frac{1}{18}}$

Section 5.5: U-Substitution Andrew Mulhausen

1. $\int (3x-2)^{20}dx$

$u = 3x-2$

$du = 3dx$

$\frac{du}{3} = dx$

$\frac{du}{3}\int u^{20}$

$\frac{du}{3}\,\frac{u^{21}}{21}$

$\frac{(3x-2)^{21}}{63} + C$

2. $\int (x^2+1)(x^3+3x)^4\,dx$

$u = x^3+3x$

$du = 3x^2+3\,dx$

$du = 3(x^2+1)\,dx$

$\frac{du}{3} = (x^2+1)\,dx$

$\frac{du}{3}\int u^4$

$\frac{du}{3}\,\frac{u^5}{5}$

$\frac{(x^3+3x)^5}{15} + C$

1. $\int_0^\pi t \sin 3t \, dt$ $\quad\left|\begin{array}{ll} u = t & dv = \sin 3t \, dt \\ du = dt & v = \dfrac{-\cos 3t}{3} \end{array}\right.$

$$= \frac{-t\cos 3t}{3} - \int \frac{-\cos 3t}{3} \, dt$$

$$= -\frac{t\cos 3t}{3} + \frac{\sin 3t}{9} \Big]_0^\pi$$

$$= \left(-\frac{\pi \cos 3\pi}{3} + \frac{\sin 3\pi}{9} \right) - 0$$

$$= \pi/3$$

Nick Watson

5.6

2. $\displaystyle\int_0^1 \frac{x}{e^{2x}}$

$u = x \qquad dv = e^{-2x}$

$du = dx \qquad v = \frac{-1}{2}e^{-2x}$

$$= \frac{-x\,e^{-2x}}{2} - \int \frac{-1}{2} e^{-2x}\,dx$$

$$= \frac{-x}{2e^{2x}} - \frac{1}{4e^{2x}}\Big]_0^1$$

$$= \frac{-1}{2e^2} - \frac{1}{4e^2} - \left(\frac{-1}{4}\right)$$

$$= \frac{1}{4} - \frac{3}{4}e^{-2}$$

Nick Watson

5.6

Emily McCue · Section 2 · PARTIAL FRACTIONS

[SOLUTIONS]

5.7

1] $\int \frac{5x-4}{2x^2+x-1}\,dx$

$\longrightarrow$ Notice: $\frac{5x-4}{2x^2+x-1} = \frac{5x-4}{(x+1)(2x-1)} \longrightarrow \frac{A}{x+1} + \frac{B}{2x-1}$

To Find A & B...

$5x-4 = A(2x-1) + B(x+1) = (2A+B)x + (-A+B)$

* Use left side of equation $(5x-4)$ to help solve for A & B...

$2A+B=5$ $\qquad$ $-A+B=4$

$A=3$ $\qquad$ $B=-1$

$\longrightarrow$ So $\frac{5x-4}{(x+1)(2x-1)} = \frac{3}{x+1} - \frac{1}{2x-1}$

Now We Integrate...

$\int \left(\frac{3}{x+1} - \frac{1}{2x-1}\right)dx = 3\ln|x+1| - \frac{1}{2}\ln|2x-1| + C$

Emily McCure · Section 2 · PARTIAL FRACTIONS

[SOLUTIONS]

2] $$\int \frac{3x^3-x^2+19x-9}{x^4+18x^2+81}\,dx$$

$\longrightarrow$ Notice: $$\frac{3x^3-x^2+19x-9}{x^4+18x^2+81} = \frac{3x^3-x^2}{x^2+9} + \frac{19x-9}{(x^2+9)^2} \rightarrow \frac{Ax+B}{x^2+9} + \frac{Cx+D}{(x^2+9)^2}$$

To Find A, B, C, & D...

$$3x^3-x^2+19x-9 = \frac{Ax+B}{x^2+9}(x^2+9)^2 + \frac{Cx+D}{(x^2+9)^2}(x^2+9)^2$$

$$= (Ax+B)(x^2+9) + (Cx+D)$$

$$= Ax^3 + Bx^2 + (9A+C)x + (9B+D)$$

* use left side values $(3x^3-x^2+19x-9)$ to solve for A, B, C, & D...

$A=3$ $\quad B=-1$ $\quad 9A+C=19$ $\quad 9B+D=-9$

$\quad C=-8$ $\quad D=0$

$\longrightarrow$ So $$\frac{3x^3-x^2+19x-9}{(x^2+9)(x^2+9)^2} = \frac{3x-1}{x^2+9} + \frac{-8x}{(x^2+9)^2}$$

Now we Integrate...

$$\int\left(\frac{3x-1}{x^2+9} + \frac{-8x}{(x^2+9)^2}\right)dx = \frac{4}{x^2+9} + \frac{3}{2}\ln|x^2+9| - \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$$

Toffee Jiaravanon

5.7 Partial Fractions - solutions

1. $$\frac{1}{x^3+2x^2+x} = \frac{1}{x(x^2+2x+x)} = \frac{1}{x(x+1)^2}$$

$$1 = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

2. $$\int \frac{2x^2+5}{(x^2+1)(x^2+4)}\,dx$$

$$\frac{2x^2+5}{(x^2+1)(x^2+4)} = \frac{A}{x^2+1} + \frac{B}{x^2+4}$$

$$\begin{aligned} 2x^2+5 &= A(x^2+4) + B(x^2+1) \\ &= Ax^2+4A + Bx^2+B \\ &= (A+B)x^2 + (4A+B) \end{aligned}$$

$(A+B)x^2 = 2x^2$ and $4A+B = 5$

$4A = 5-B$

$A+B = 2$

so $A=1$ and $B=1$, so

$$\int \frac{2x^2+5}{(x^2+1)(x^2+4)}\,dx = \int\left(\frac{1}{x^2+1} + \frac{1}{x^2+4}\right)dx$$

$$= \ln|x^2+1| + \ln|x^2+4| + C$$

$\cancel{1) \int (\sin x)^2 (\cos x)^3 \, dx}$

Chelsea Marshall
TRIG INTEGRALS

5.7

$\cancel{2) \int \cos^2 x \, dx}$

SOLUTIONS

SOLUTIONS

1) $\int \sin^2 x \cos^3 x \, dx$

- the one pulled out must start with an odd power

$= \int \sin^2 x \cos^2 x \cos x$

$u = \sin x \quad du = \cos x \, dx$

$\cos^2 x = 1 - \sin^2 x$

$= \int u^2 (1 - u^2) \, du$

$= \int (u^2 - u^4) \, du$

$= \frac{1}{3} u^3 - \frac{1}{5} u^5 + C = \boxed{\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C}$

2) $\int \cos^2 x \, dx$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \cos^2 x - (1 - \cos^2 x)$

$\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{\cos 2x + 1}{2}$

$= \int \frac{\cos 2x + 1}{2}$

$= \int \frac{\cos 2x}{2} + \frac{1}{2}$

$\boxed{= \frac{1}{2} x + \frac{1}{4} \sin 2x + C}$

Daniela silva
Section 5.7
16. trig. Substituti

16. $\displaystyle\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}\,dx$

$x = \sin\theta$
$x = 4\sin\theta$
$dx = 4\cos\theta\, d\theta$

$$\int \frac{(4\sin\theta)^3 \cdot 4\cos\theta\, d\theta}{\sqrt{16-(4\sin\theta)^2}}$$

$$\int \frac{4^4 \sin^3\theta \cdot \cos\theta\, d\theta}{\sqrt{4^2 - 4^2 \cdot \sin^2\theta}}$$

$$\int \frac{4^4 \sin^3\theta \cdot \cos\theta\, d\theta}{\sqrt{4^2(1-\sin^2\theta)}}$$

$$\int \frac{4^4 \sin^3\theta \cdot \cos\theta\, d\theta}{4\sqrt{(1-\sin^2\theta)}}$$

$$= 4^3 \int \frac{\sin^3\theta \cdot \cos\theta}{\sqrt{\underbrace{(1-\sin^2\theta)}_{\cos^2\theta}}}\, d\theta$$

$$= 4^3 \int \frac{\sin^3\theta \cdot \cos\theta}{\sqrt{\cos^2\theta}}\, d\theta$$

$$= 4^3 \int \frac{\sin^3\theta \cdot \cancel{\cos\theta}}{\cancel{\cos\theta}}\, d\theta$$

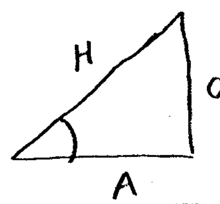
$$= 4^3 \int \sin^3\theta$$

$$4^3 \int (\sin^2\theta)(\sin\theta)$$

$u = \cos\theta$
$\frac{1}{\sin u} du = \frac{\sin\theta}{\cancel{\sin\theta}} d\theta$

$$4^3 \int (1-\cos^2\theta)(\sin\theta)$$

$$4^3 \int (1-u^2)$$



$C = \frac{A}{H} = \frac{\sqrt{16-x^2}}{4}$ $\quad S = \frac{O}{H} = \frac{x}{4}$

$$= 4^3 \left[u - \frac{u^3}{3}\right]$$

$$4^3 \left[\cos\theta - \frac{(\cos\theta)^3}{3}\right]$$

$$= 4^3 \left[\frac{\sqrt{16-x^2}}{4} - \frac{1}{3}\cdot\left(\frac{\sqrt{16-x^2}}{4}\right)^3\right]_0^{2\sqrt{3}}$$

$$= 4^3 \cdot \left[\frac{\sqrt{10}}{4} - \frac{1}{3}\cdot\frac{5\sqrt{10}}{32}\right]_0^{2\sqrt{3}}$$

$$= \frac{38\sqrt{10}}{3}$$

≈ 40.05

Daniela Silva
Section 5.7 trig. substitution

18.

$$\int \frac{x^3}{\sqrt{x^2+1}}\,dx$$

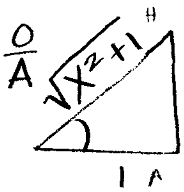
$x = a\tan\theta$

$x = \tan\theta$

$dx = \sec^2\theta$

$1+\tan^2\theta = \sec^2\theta$

$\tan\theta = \frac{O}{A}$



$\cos = \frac{A}{H}$

$\frac{x}{1} = \tan\theta$

$$\int \frac{(\tan\theta)^3}{\sqrt{(\tan\theta)^2+1}} \cdot \sec^2\theta$$

$$\int \frac{(\tan\theta)^3 \cdot \sec^2\theta}{\sqrt{(\tan\theta+1)}}\,d\theta$$

$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{x^2+1}}{1}$

$$\int \frac{(\tan\theta)^3 \cdot \sec^2\theta}{\sqrt{\sec^2\theta}}\,d\theta$$

$$= \int \frac{(\tan\theta)^3 \cdot \sec^{\cancel{2}}\theta}{\cancel{\sec\theta}}\,d\theta$$

$$= \int (\tan\theta)^3 \cdot \sec\theta \, d\theta$$

$$= \int (\tan^2\theta) \tan\theta \cdot \sec\theta \, d\theta$$

$$= \int (\sec^2\theta - 1) \cdot \tan\theta \cdot \sec\theta \, d\theta$$

$u = \sec\theta$

$du = \sec\theta\tan\theta \, d\theta$

$d\theta = \frac{1}{\sec\theta\tan\theta}$

$$\int (u^2-1)\,du$$

$$= \frac{u^3}{3} - u + C$$

$$= \frac{(\sec\theta)^3}{3} - \sec\theta + C$$

$$= \boxed{\frac{(\sqrt{x^2+1})^3}{3} - \sqrt{x^2+1} + C}$$

Alix Naugler
Math 151-2

Solutions to Trig Substitution Problems (5.7)

1. $$\int \frac{dx}{x^2\sqrt{x^2+4}}$$

$a=2$
$x=2\tan\theta$
$dx=2\sec^2\theta$

$\tan^{-1}\left(\frac{x}{2}\right)=\theta$

$$\int \frac{2\sec^2\theta\, d\theta}{(2\tan\theta)^2\sqrt{4\tan^2\theta+4}}$$

$$\int \frac{2\sec^2\theta\, d\theta}{(4\tan^2\theta)(2\sec\theta)}$$

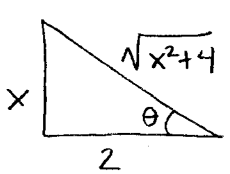
$$\int \frac{\sec\theta\, d\theta}{4\tan^2\theta}$$

$$\frac{1}{4}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos\theta}}\, d\theta = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

$u=\sin\theta$
$du=\cos\theta\, d\theta$

$$\frac{1}{4}\int \frac{1}{u^2}\, du = \frac{1}{4}\left(\frac{-1}{u}\right)+C$$

$$= -\frac{1}{4\sin\theta}+C$$



$\tan^{-1}\left(\frac{x}{2}\right)=\theta$

$\sin\theta = \frac{x}{\sqrt{x^2+4}}$

$$= -\frac{1}{4}\left(\frac{x}{\sqrt{x^2+4}}\right)+C$$

2. $$\int \frac{\sqrt{9-x^2}}{x^2}\, dx$$

$a=3$
$x=3\sin\theta$
$dx=3\cos\theta\, d\theta$

$$\int \frac{\sqrt{9-(3\sin\theta)^2}\cdot 3\cos\theta\, d\theta}{(3\sin\theta)^2}$$

$$\int \frac{\sqrt{9-9\sin^2\theta}\cdot 3\cos\theta\, d\theta}{9\sin^2\theta}$$

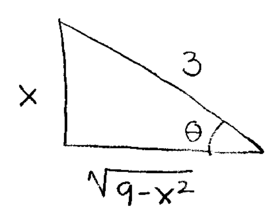
$$\int \frac{\sqrt{(1-\sin^2\theta)9}\cdot 3\cos\theta\, d\theta}{9\sin^2\theta}$$

$$\int \frac{3\cos\theta\,(3\cos\theta\, d\theta)}{9\sin^2\theta}$$

$$\int \frac{9\cos^2\theta}{9\sin^2\theta} = \int \cot^2\theta\, d\theta$$

$$= \int \csc^2\theta - 1\, d\theta$$

$$= -\cot\theta - \theta + C$$



$\sin^{-1}\left(\frac{x}{3}\right)=\theta$

$\cot\theta = \frac{\sqrt{9-x^2}}{x}$

$$= -\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right)+C$$

$\Downarrow$

$\theta=\sin^{-1}\left(\frac{x}{3}\right)$

Alisa Mackesy
5.8

Tables of Integrals

1) $\int x^3 \sin x \, dx$

use T1: $\int u\,dv = uv - \int v\,du$

Let $u = x^3$ $\quad dv = \sin x \, dx$

$du = 3x^2 dx$ $\quad v = -\cos x$

$= 3\int x^2 \cos x \, dx - x^3 \cos x$

use T1

Let $u = x^2$ $\quad dv = \cos x \, dx$

$du = 2x dx$ $\quad v = \sin x$

$= x^3(-\cos x) + 3x^2 \sin x - 6\int x \sin x \, dx$

use T1

Let $u = x$ $\quad dv = \sin x \, dx$

$du = dx$ $\quad v = -\cos x$

$= x^3(-\cos x) + 3x^2 \sin x + 6x\cos x - 6\int \cos x \, dx$

$= x^3(-\cos x) + 3x^2 \sin x - 6\sin x + 6x\cos x + C$

$= 3(x^2 - 2)\sin x - x(x^2 - 6)\cos x + C$

Tables of Integrals

Alisa Mackey
5.8

2) $\int \tan^3(\pi x)\,dx$

Let $u = \pi x$

$du = \pi\,dx$

$\int \tan^3(\pi x)\,dx = \frac{1}{\pi}\int \tan^3(u)\,du$

use T75: $\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1}u - \int \tan^{n-2} u\,du$

where $n=3$

$= \frac{\tan^2 u}{2\pi} - \frac{1}{\pi}\int \tan(u)\,du$

remember: $\tan(u) = \frac{\sin(u)}{\cos(u)}$

$= \frac{\tan^2 u}{2\pi} - \frac{1}{\pi}\int \underbrace{\frac{\sin(u)}{\cos(u)}\,du}$

let $s = \cos(u)$

$ds = -\sin(u)\,du$

$= \frac{\tan^2(u)}{2\pi} - \frac{1}{\pi}\int -\frac{1}{s}\,ds$

$= \frac{1}{\pi}\int \frac{1}{s}\,ds + \frac{\tan^2(u)}{2\pi}$

$= \frac{\ln(s)}{\pi} + \frac{\tan^2(u)}{2\pi} + C$

$= \frac{\frac{\tan^2(u)}{2} + \ln(\cos(u))}{\pi} + C$

$= \frac{\frac{1}{2}\tan^2(\pi x) + \ln(\cos(\pi x))}{\pi} + C$

Alisa Mackesy
5.8

$$= \frac{\tan^2(\pi x) + 2\ln(\cos(\pi x))}{2\pi} + C$$

$$= \boxed{\frac{\sec^2(\pi x)}{2\pi} + \frac{\ln(\cos(\pi x))}{\pi} + C}$$ Solution to ~~#~~ 2

(tables of Integrals)

15. $\int_1^5 \frac{\cos x}{x}\, dx \quad n=8 \qquad \frac{5-1}{8} = \frac{1}{2}$ Sec 5.9

$\frac{1}{2} \cdot \frac{1}{2} \Big[\frac{\cos(1)}{1} + 2\left(\frac{\cos(1.5)}{1.5}\right) + 2\left(\frac{\cos(2)}{2}\right) + 2\left(\frac{\cos(2.5)}{2.5}\right) +$

$2\left(\frac{\cos(3)}{3}\right) + 2\left(\frac{\cos(3.5)}{3.5}\right) + 2\left(\frac{\cos 4}{4}\right) + 2\left(\frac{\cos 4.5}{4.5}\right)$

$+ \left(\frac{\cos 5}{5}\right) \Big] =$

$\frac{1}{4}$ (.5403 + .09432 - .4161 - .6409 - .66 - .5351 - .3268 - ...

... 3125

M.P.

$\frac{1}{2} \Big[\left(\frac{\cos(1.25)}{1.25}\right) + \frac{\cos(1.75)}{1.75} + \frac{\cos(2.25)}{2.25} + \frac{\cos(2.75)}{2.75} +$

$\frac{\cos(3.25)}{3.25} + \frac{\cos(3.75)}{3.75} + \frac{\cos(4.25)}{4.25} + \frac{\cos(4.75)}{4.75} \Big] =$

$\frac{1}{2}$ (.2526 - .1019 - .2792 - .3361 - .3059 - .2158 - .105 + .00...

= -.54336

Answers: comparison ~~test~~ for integrals
section (5.10) p 420

Alexa Villaseñor

43. $\int_0^\infty \frac{x}{x^3+1}\,dx$

$\underbrace{\int_0^\infty \frac{x}{x^3+1}dx}_{g(x)} \le \overbrace{\int_0^\infty \frac{x}{x^3}dx}^{f(x)}$

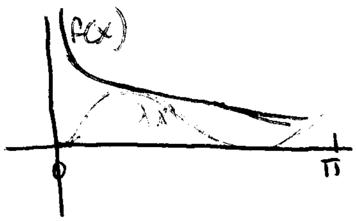
$= \int_0^\infty \frac{1}{x^2}dx$ ← p-series - $2>1$
F(x) converges

* according to p-series test, f(x) converges
So since $\int_0^\infty f(x)$ converges, then $\int_0^\infty g(x)$ converges.

48. $\int_0^\pi \frac{\sin^2 x}{\sqrt{x}}\,dx$

$\underbrace{\int_0^\pi \frac{\sin^2 x}{\sqrt{x}}dx}_{g(x)} \le \overbrace{\int_0^\pi 1/\sqrt{x}\,dx}^{f(x)}$

$= x^{-1/2}$



$\int_0^\pi x^{-1/2}$

$= \lim_{t\to 0} \int_t^\pi x^{-1/2}\,dx$

$= \lim_{t\to 0} 2x^{1/2}\Big|_t^\pi$

$\lim_{t\to 0}\left(2\pi^{1/2} - 2t^{1/2}\right)$

$= 2\pi^{1/2} - 2(0)^{1/2}$

$= 2\pi^{1/2}$

↳ since the number is a real number that exist, the integral converges.

$\int_0^\pi f(x)$ converges, then $\int_0^\pi g(x)$ converges as well.

#1: $\int_1^{\infty} \frac{dx}{(5x+2)^2}$ Answer: Improper Integrals Sara Aranda

5.10

① convert to proper integral: $\lim_{n \to \infty} \int_1^n \frac{dx}{(5x+2)^2}$

② use u substitution:

$u = 5x+2$

$du = 5dx$

$\frac{du}{5} = dx$

$$\int_1^N \frac{dx}{(5x+2)^2} = \frac{1}{5}\int_1^{2N+5} \frac{du}{u^2}$$

$$= \frac{1}{5}\int_7^{5N+2} u^{-2} du$$

$$= \frac{1}{5}(-u^{-1})\Big|_7^{5n+2}$$

$$= \frac{1}{5}\left(-\frac{1}{5n+2} - \left(-\frac{1}{7}\right)\right)$$

$$= \frac{1}{35} - \frac{1}{5(5n+2)}$$

③ evaluate limit:

$$\int_1^{\infty} \frac{dx}{(5x+2)^2} = \lim_{n \to \infty} \int_1^n \frac{dx}{(5x+2)^2}$$

$$= \lim_{n \to \infty} \left(\frac{1}{35} - \frac{1}{25n+10}\right)$$

$$= \frac{1}{35} - 0 \approx .03$$

#2 $\int_0^1 \frac{\ln x}{x^{1/2}}\,dx$ Answer: Improper integral 5.10

int by parts $\int_0^1 \frac{\ln x}{x^{1/2}}\,dx$

$u = \ln x \quad u' = \frac{1}{x}$

$v = 2x^{1/2} \quad v' = x^{-1/2}$

$$2x^{1/2}\ln x\Big|_0^1 - 2\int_0^1 x^{-1/2}\,dx$$

$$2x^{1/2}\ln x\Big|_0^1 - 4x^{1/2}\Big|_0^1$$

take limit

$$\lim_{b\to0^+} 2b^{1/2}\ln b$$

$$\lim_{b\to0^+} \frac{2\ln b}{b^{-1/2}}$$

L'Hopitals Rule

$$\lim_{b\to0^+} \frac{\frac{2}{b}}{-\frac{1}{2}b^{-3/2}}$$

$$\lim_{b\to0^+} -4b^{1/2} = 0$$

plug back in: $2x^{1/2}\ln x\Big|_0^1 - 4$

$$0 - 4$$

$$\boxed{= -4}$$

5.10

Xiaoye Yang

Improper Integrals

(1) $\int_{-\infty}^{\infty} x e^{-x^2} dx$

$= \lim_{B\to\infty} \int_{-B}^{B} x e^{-x^2} dx$

$= \lim_{B\to\infty} -\frac{1}{2} e^{-x^2} \Big|_{-B}^{B}$

$= \lim_{B\to\infty} -\frac{1}{2} e^{-B^2} + \frac{1}{2} e^{-(-B)^2}$

$= \lim_{B\to\infty} -\frac{1}{2} e^{-B^2} + \frac{1}{2} e^{-B^2}$

$= \lim_{B\to\infty} 0$

$= 0$

(2) $\int_{0}^{33} (x-1)^{-\frac{1}{5}} dx$

$= \lim_{B\to 1} \int_{0}^{B} (x-1)^{-\frac{1}{5}} dx + \int_{B}^{33} (x-1)^{-\frac{1}{5}} dx$

$= \lim_{B\to 1} \frac{5}{4} (x-1)^{\frac{4}{5}} \Big|_{0}^{B} + \frac{5}{4} (x-1)^{\frac{4}{5}} \Big|_{B}^{33}$

$= \lim_{B\to 1} \frac{5}{4} (B-1)^{\frac{4}{5}} - \frac{5}{4} (-1)^{\frac{4}{5}} + \frac{5}{4} (32)^{\frac{4}{5}} - \frac{5}{4} (B-1)^{\frac{4}{5}}$

$= \lim_{B\to 1} -\frac{5}{4} (+1) + \frac{5}{4} \cdot 16$

$= \lim_{B\to 1} -\frac{5}{4} + 20$

$= \lim_{B\to 1} \frac{75}{4}$

$= \frac{75}{4}$

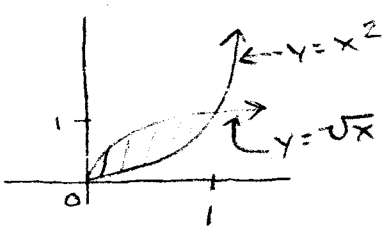
Matt Laurent Sect. 2

6.1 Solutions

(Area bounded by $y=x^2$ & $y^2=x$)

① • Decide whether you want to solve the problem with respect to x or y

• we'll do so with respect to x so through basic algebra we find $y=x^2$ & $y=\sqrt{x}$



$$\int_0^1 \sqrt{x} - x^2 \, dx$$

$$\frac{2}{3}x^{3/2} - \frac{x^3}{3}\Big|_0^1$$

$$(2/3 - 1/3) - (0-0)$$

$$\boxed{1/3}$$

② Find $V_K - V_C$

t	0	1	2	3	4	5	6	7	8	9	10
$V_K - V_C$	0	2	5	6	7	9	11	11	12	12	12

Now use Simpsons rule $\frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$$1/3\,[0 + 4(2) + 2(5) + 4(6) + 2(7) + 4(9) + 2(11) + 4(11) + 2(12) + 4(12) + 12]$$

80.6667 miles

★ if you want it in feet as it is in the back of the book, remember $1\,mi/h = \frac{5280}{3600}\,ft/s$

so $\frac{80.667}{1} \cdot \frac{5280}{3600} \approx 118$ ft

Metaeb Alohali 6.1

1.

To find the aner we to inegrate $\int$ top curve − buttom curve with respect to x from 0 to 4

$$\int_0^4 (5x-x^2)-(x)\,dx$$

$$=\int_0^4 5x-x^2-x\,dx$$

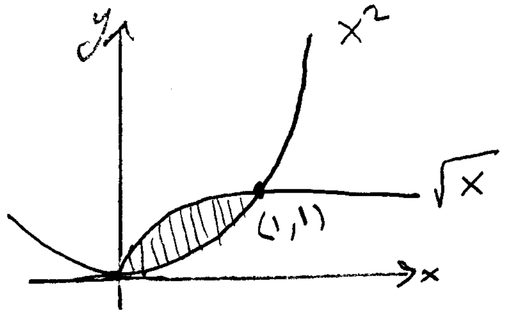
$$=\frac{5x^2}{2}-\frac{x^3}{3}-\frac{x^2}{2}\Big|_0^4$$

$$=\left[\frac{5(4)^2}{2}-\frac{4^3}{3}-\frac{4^2}{2}\right]-0$$

$$=\frac{80}{2}-\frac{64}{3}-\frac{16}{2}$$

$$=\boxed{\frac{32}{3}}$$

2.



$y^2=x$

$y=\sqrt{x}$

$$\int_0^1 (\sqrt{x})-(x^2)\,dx$$

$$=\int_0^1 x^{\frac{1}{2}}-x^2\,dx$$

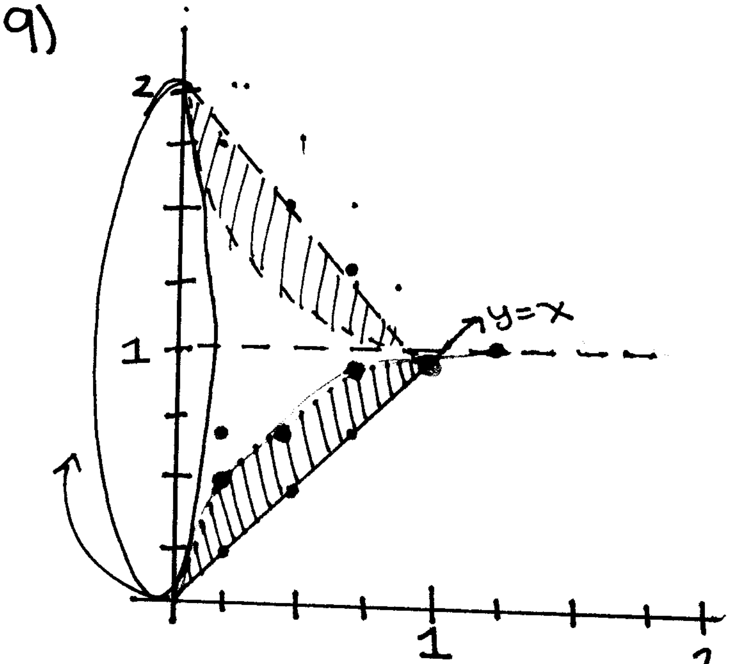
$$=\frac{x^{3/2}}{3/2}-\frac{x^3}{3}\Big|_0^1$$

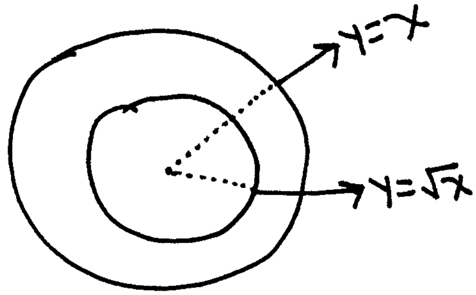
$$=\left[\frac{2}{3}(1)^{3/2}-\frac{(1)^3}{3}\right]-0$$

$$=\frac{2}{3}-\frac{1}{3}=\boxed{\frac{1}{3}}$$

6.2- Meredith Hoggatt
INTEGRALS & VOLUME
SOLUTIONS

9)





~~Area of washer = $\pi x^2 - \pi x$~~

centered @ y=1

Area of washer: $\pi(1-x)^2 - \pi(1-\sqrt{x})^2$

$= \pi(1-2x+x^2) - \pi(1-2\sqrt{x}+x)$

$= \pi - 2\pi x + \pi x^2 - \pi + 2\pi\sqrt{x} - \pi x$

$= -3\pi x + 2\pi\sqrt{x} + \pi x^2$

Volume washer $= (-3\pi x + 2\pi\sqrt{x} + \pi x^2)\Delta x$

Volume solid:

$$\int_0^1 -3\pi x + 2\pi\sqrt{x} + \pi x^2 \, dx$$

$$= \pi \int_0^1 -3x + 2\sqrt{x} + x^2 \, dx$$

$$= \pi \left[\frac{-3}{2}x^2 + \frac{4}{3}x^{3/2} + \frac{1}{3}x^3 \right]_0^1$$

$$= \pi \left(\frac{-3}{2}(1) + \frac{4}{3}(1) + \frac{1}{3}(1) \right)$$

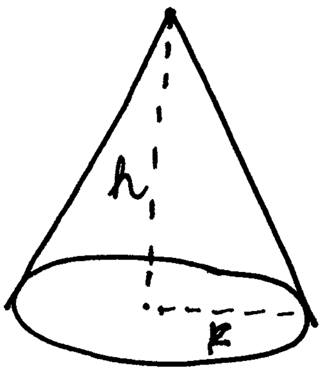
$$= \pi \left(\frac{1}{6} \right)$$

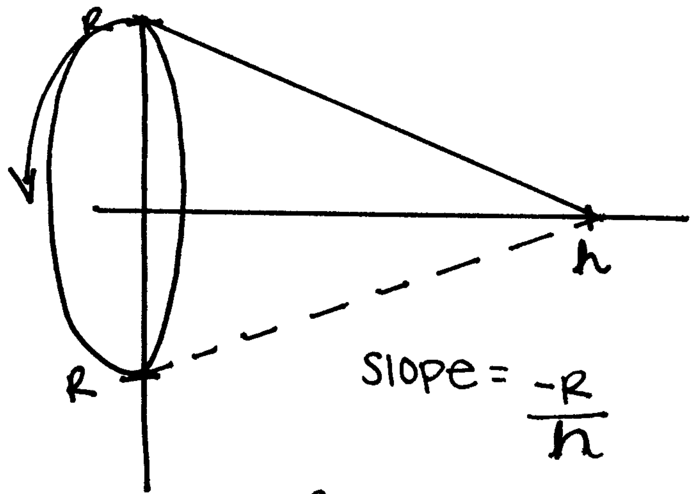
$$= \frac{\pi}{6}$$

6.2 - Meredith Hoggatt

INTEGRALS & VOLUME SOLUTIONS

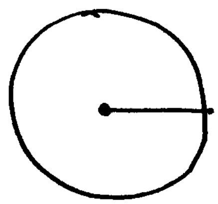
31.





slope $= \frac{-R}{h}$

$$y = \frac{-R}{h}x + R$$



Radius disk $= -\frac{R}{h}x + R$

Area Disk $= \pi\left(-\frac{R}{h}x + R\right)^2$

Volume Disk $= \pi\left(-\frac{R}{h}x + R\right)^2 \Delta x$

Volume Cone:

$$\int_0^h \pi\left(\frac{-R}{h}x + R\right)^2 dx$$

$$= \pi \int_0^h \frac{R^2}{h^2}x^2 \text{ [redacted] } 2\frac{R^2}{h}x + R^2 \, dx$$

$$= \pi\left(\frac{R^2}{3h^2}x^3 - \frac{R^2}{h}x^2 + R^2 x \Big|_0^h\right.$$

$$= \pi\left(\frac{R^2}{3h^2}(h^3) - \frac{R^2}{h}(h^2) + R^2(h)\right)$$

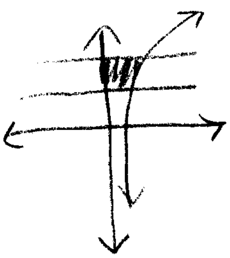
$$= \pi\left(\frac{R^2 h}{3} - R^2 h + R^2 h\right)$$

$$= \frac{1}{3}\pi R^2 h$$

6.2: Volumes

Find the volume of the solid obtained by rotating the region bounded by the given curves.

1. $y = \ln x,\ y = 1,\ y = 2,\ x = 0$; about y-axis



$y = \ln x \quad x = e^y$

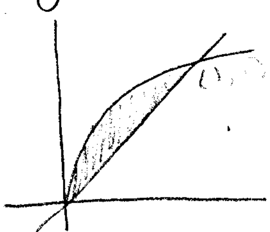
$r = e^y$

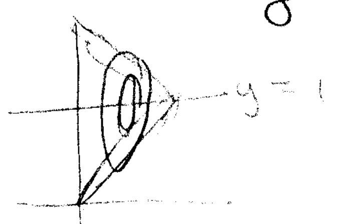
$A = \pi (e^y)^2 \Delta y$

$V = \pi \int_1^2 (e^{2y})\,dy$

$\frac{\pi}{2} e^{2y} \Big|_1^2 = \boxed{\frac{\pi}{2}(e^4 - e^2)}$

2. $y = x,\ y = \sqrt{x}$; about $y = 1$





$R = 1 - x$

$r = 1 - \sqrt{x}$

$1 - 2x + x^2 - 1 + 2\sqrt{x} - x$

$\pi \int_0^1 (-3x + 2x^{1/2} + x^2)\,dx$

$\pi \left(-3x^2/2 + 4x^{3/2}/3 + x^3/3\right) \Big|_0^1$

$\pi(-3/2 + 4/3 + 1/3) = \boxed{\pi/6}$

Section 6.4: Arc Length (Solutions)

Rachel Lloyd

$$\text{arc length} = \int_a^b \sqrt{(f'(x))^2 + 1}$$

1) $x = y^{3/2}$, $0 \le y \le 1$

$$\int_0^1 \sqrt{\left(\tfrac{3}{2}\sqrt{y}\right)^2 + 1}\, dy = \int_0^1 \sqrt{\tfrac{9}{4}y + 1}\, dy = \int_0^1 \left(\tfrac{9}{4}y + 1\right)^{1/2} dy$$

$$= \frac{2}{3}\left(\frac{9}{4}y + 1\right)^{3/2} \cdot \frac{4}{9} \Big|_0^1 = \frac{8}{27}\left(\frac{9}{4}y + 1\right)^{3/2} \Big|_0^1$$

$$= \frac{8}{27}\left(\frac{9}{4}(1) + 1\right)^{3/2} - \frac{8}{27}\left(\frac{9}{4}(0) + 1\right)^{3/2} = \frac{8\left(\frac{13}{4}\right)^{3/2} - 8}{27} = \frac{8\left(\sqrt{\frac{13^3}{4^3}}\right) - 8}{27}$$

$$= \frac{8\left(\frac{\sqrt{2197}}{8}\right) - 8}{27} = \boxed{\frac{13\sqrt{13} - 8}{27}}$$

2) $\sin\left(\frac{\pi x}{7}\right)$, $0 \le x \le 28$

$$\int_0^{28} \sqrt{\left(\tfrac{\pi}{7}\cos\left(\tfrac{\pi x}{7}\right)\right)^2 + 1}\, dx = \int_0^{28} \sqrt{\tfrac{\pi^2}{49}\cos^2\left(\tfrac{\pi x}{7}\right) + 1}\, dx \approx \boxed{29.36}$$

use calculator

Section 6.5 Solution

Jacqueline Mai[illegible]
Section 2

4. $f(\theta) = \sec^2\left(\frac{\theta}{2}\right) \quad \left[0, \frac{\pi}{2}\right]$

$$= \frac{1}{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sec^2\left(\frac{\theta}{2}\right) d\theta$$

$$= \frac{2}{\pi}\left[\tan\left(\frac{\theta}{2}\right) \cdot 2 \Big|_0^{\frac{\pi}{2}}\right]$$

$$= \frac{4}{\pi}\left[\tan\left(\frac{\pi}{4}\right) - \tan(0)\right]$$

$$= \frac{4}{\pi}\left[1 - 0\right] = \boxed{\frac{4}{\pi}}$$

16. $T(t) = 20 + 75e^{-t/50} \quad [0, 30]$

$$= \frac{1}{30}\int_0^{30} 20 + 75e^{-t/50}$$

$$\frac{1}{30}\left[20t + 75e^{-t/50}(50)\Big|_0^{30}\right]$$

$$= \frac{1}{30}\left(20t - 3750e^{-t/50}\right)\Big|_0^{30}$$

$$= \frac{1}{30}\left(600 - 3750e^{\frac{-30}{50}} - (-3750e^{0})\right)$$

$$= \frac{1}{30}\left(600 - 2058.0436 + 3750\right)$$

$$\approx 76.3985^\circ C$$

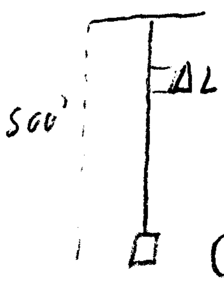
$$\approx \boxed{76^\circ C}$$

Mark Hupp
Section 6
Work (non liquid)

Problem # 13(p473)
-A cable that weighs 2lb/ft is used to lift 800lbs of coal up a mine shaft 500ft deep. Find the work done.

800 Lbs Coal

$2\Delta L$ Lbs cable



$$\int_{500}^{0} 800 + 2L\,dL = 800L + \frac{2L^2}{2}\Big|_{500}^{0}$$

$$0 - \left(800(500) + \frac{\cancel{2}(500)^2}{\cancel{2}}\right) = W = 650{,}000 \text{ FT Lbs}$$

Problem #27 (p474)
a) Newtons Law ogg Gravitation states two bodies with masses m1,m2 attract each other with a force

$$= \frac{G(m1)(m2)}{r^2}$$

where r is the distance between the bodies, and G is a gravitational constant. If one of the bodies is fixed, find the work needed to move the other from r=a to r=b

$$\int_a^b G\frac{(m_1)(m_2)}{r^2}\,dr = G(m_1)(m_2)\left(-\frac{1}{r}\Big|_a^b\right) = G(m1)(m_2)\left(\frac{1}{a}-\frac{1}{b}\right)$$

b)Find the work required to launch a 1000Kg satellite 1000Km vertically into orbit.
Assume the earths mass =5.98x10^24Kg, radius of the earth =6.37x10^6 and G =6.67x10^-11$\frac{\text{Nm}^2}{\text{Kg}^2}$

$$G(m_1)(m_2) = (6.67\times10^{-11})(1000)(5.98\times10^{24}) = 3.989\times10^{17}$$

$$a = 6.37\times10^6\,m$$

$$b = 6.37\times10^6\,m + 1\times10^6\,m = 7.73\times10^6$$

$$\int_{6.37\times10^6}^{7.37\times10^6} \frac{3.989\times10^{17}}{r^2}\,dr = 3.989\times10^{17}\left(\frac{1}{6.37\times10^6} - \frac{1}{7.37\times10^6}\right)$$

$$= 8496833657 \approx 8.5\times10^9$$

(13) weight of cable = 2 lb/ft

Weight of coal = 800 lb

Distance = 500 ft

$$W = \int_0^{500} \overset{\text{cable}}{2x\,dx}$$

$$= x^2\Big|_0^{500} = 250000 \text{ ft-lb}$$

[illegible]

W_{c} = 800 × 500

= [illegible],000 ft-lb

Total work done

W = W_c [illegible]

= [illegible] + 400,000

$\boxed{= 650{,}000 \text{ ft-lb}}$

(16) A) $\frac{1}{2}\Delta x\, dx$

$$W = \int_0^{50} \frac{1}{2} x\, dx$$

$$= \frac{1}{4}\left[x^2\right]_0^{50} = \frac{1}{4}(2500)$$

$\boxed{= 625 \text{ ft-lb}}$

B) $W = \int_2 \frac{x}{2}$ [illegible]

Section 6.6

Jamal

Jamal

Marc Tang

$p(x)$: demand curve.
x: units sold
P = price

Consumer surplus 6.7

$$\int_0^X [p(x) - P]\,dx$$

1.) $p(x) = 450/x+8$

$P = 10$

$$10 = \frac{450}{x+8}$$

$10x + 80 = 450$

$10x = 370$; $x = 37$ units sold

$$\int_0^{37} \frac{450}{x+8} - 10\,dx;\ 450\ln|x+8| - 10x \Big|_0^{37}$$

$$450\ln 45 - 370 - (450\ln 8 - 0)$$

$\approx \$407.75$

2.) $p(x) = 2000 - 0.1x - 0.01x^2$

sales level = 100

$P = 2000 - 0.1(100) - 0.01(100^2)$

$P = 2000 - 10 - 100$

$P = 1890$

$$\int_0^{100} 2000 - 0.1x - 0.01x^2 - 1890\,dx$$

$$\int_0^{100} 110 - 0.1x - 0.01x^2\,dx$$

$$110x - \frac{0.1x^2}{2} - \frac{0.01x^3}{3} \Big|_0^{100}$$

$11000 - 500 - 3,333.33\overline{3}$

$\approx \$7166.67$

[redacted]

8.1

Connor Stearns

11) $a_n = \dfrac{3+5n^2}{n+n^2}$

$$\lim_{n\to\infty} \frac{3+5n^2}{n^2+n} = \lim_{n\to\infty} \frac{10n}{2n+1} = \lim_{n\to\infty} \frac{10}{2} = \boxed{5}$$

$\boxed{\therefore \text{ It converges}}$

50) $a_n = \dfrac{2n-3}{3n+4}$

$$a_{n+1} = \frac{2(n+1)-3}{3(n+1)+4}$$

$$= \frac{2n+2-3}{3n+3+4}$$

$$= \frac{2n-1}{3n+7}$$

$$\lim_{n\to\infty} \frac{2n-3}{3n+4} = \frac{2}{3}$$

So, when $n=1$

$$a_n = \frac{2(1)-3}{3(1)+4} = \frac{-1}{7} \qquad a_{n+1} = \frac{2(1)-1}{3(1)+7} = \frac{1}{10}$$

$\therefore$ The sequence is increasing as $\frac{1}{10} \geq -\frac{1}{7}$ and is bounded as the $\lim\limits_{n\to\infty} a_n = \frac{2}{3}$

Mohamed Al Hosani

Chapter 8.1

Answers for Review Questions

30) $a_n = \dfrac{\sin 2n}{1+\sqrt{n}}$

* The numerator of this sequence ranges from -1 to 1

$$\frac{-1}{1+\sqrt{n}} \leqslant \frac{\sin 2n}{1+\sqrt{n}} \leqslant \frac{1}{1+\sqrt{n}}$$

* take the limit as n approaches infinity on the left terms of the inequality. To do this, divide the numerator and the denominator by the highest power of n and apply limit laws

$$\lim_{n\to\infty} \frac{-1}{1+\sqrt{n}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}(-1)}{\frac{1}{\sqrt{n}}(1+\sqrt{n})} = \lim_{n\to\infty} \frac{\frac{-1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}+\frac{\sqrt{n}}{\sqrt{n}}}$$

$$= \lim_{n\to\infty} \frac{\frac{-1}{\sqrt{n}}}{\lim\limits_{n\to\infty}\frac{1}{\sqrt{n}} + \lim\limits_{n\to\infty} 1} = \frac{0}{0+1} \Rightarrow \lim_{n\to\infty} \frac{-1}{1+\sqrt{n}} = 0$$

* Apply the Previous Step on the right terms of the inequality

$$\lim_{n\to\infty} \frac{1}{1+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}(1)}{\frac{1}{\sqrt{n}}(1+\sqrt{n})} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}} + \frac{\sqrt{n}}{\sqrt{n}}}$$

$$= \frac{\lim_{n\to\infty} \frac{1}{\sqrt{n}}}{\lim_{n\to\infty} \frac{1}{\sqrt{n}} + \lim_{n\to\infty} 1} = \frac{0}{0+1} \Rightarrow \lim_{n\to\infty} \frac{1}{1+\sqrt{n}} = 0$$

* Since the limit of n as it approaches infinity on both sides of the inequality goes to zero, therefore, by the squeeze theorem

$$\lim_{n\to\infty} \frac{\sin 2n}{1+\sqrt{n}} = 0$$ Therefore the sequence is convergent.

Mohamed Al Hosani

Chapter 8.1

52) $a_n = n + \frac{1}{n}$

For $n = 1$
$a_1 = 2$

For $n = 2$
$a_2 = 2 + \frac{1}{2} = 2.5$

For $n = 3$
$a_3 = 3 + 1/3 = 3.333$

For $n = 4$
$a_4 = 4 + \frac{1}{4} = 4.25$

* Here it is clear that the terms of the given sequence are increasing, So the sequence is monotonically increasing. Also, the given sequence a_n is not bounded because there is no upper bound of the terms of sequence.

Geometric Series

Tucker Reiland

8.2

(11) $3 - 4 + \frac{16}{3} - \frac{64}{9} + \ldots$

$$\sum_{n=1}^{\infty} (-1)^{n-1} 3\left(\frac{4}{3}\right)^{n-1}$$

$r = \frac{4}{3}$

$|r| > 1$

Diverges

(13) $10 - 2 + 0.4 - 0.08 + \ldots$

$$\sum_{n=1}^{\infty} 10\left(-\frac{1}{5}\right)^{n-1}$$

$r = -\frac{1}{5}$

$|r| < 1$

$$\frac{a}{1-r} = \frac{10}{1+\frac{1}{5}} = \frac{10}{\frac{6}{5}} = \frac{50}{6} = \frac{25}{3}$$

Joylene Aguirre sec. 3
Topic: 8.2

Determine whether the series is convergent or divergent. If it is convergent, find the sum.

1.) $$\sum_{n=1}^{\infty} \frac{n^2+2n+1}{500n^2+2n+1}$$

$$\lim_{n\to\infty} \frac{n^{2}+2n+1}{500n^{2}+2n+1} = \frac{1}{500} \neq 0$$

Therefore, by our n^{th} term test for divergence, the series diverges.

2.) $$\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2}$$

first, need partial sums

$$S_n = \sum_{i=0}^{n} \frac{1}{i^2+3i+2}$$

$$\frac{1}{i^2+3i+2} = \frac{1}{(i+2)(i+1)} = \frac{1}{(i+1)} - \frac{1}{(i+2)}$$

$$S_n = \sum \left(\frac{1}{i+1} - \frac{1}{i+2}\right) = \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \dots \left(\cancel{\frac{1}{n}} - \cancel{\frac{1}{n+1}}\right) + \left(\cancel{\frac{1}{n+1}} - \frac{1}{n+2}\right)$$

$$= 1 - \frac{1}{n+2}$$ → telescoping series

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \frac{1}{n+2}\right) = 1$$

$$\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2} = 1$$

The sequence of partial sums is convergent and so the series is convergent and has a value of 1.

Geometric Series Solutions

① $$\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{\pi^n}{3 \cdot 3^n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3}\left(\frac{\pi}{3}\right)^n \quad r = \frac{\pi}{3} \quad \left|\frac{\pi}{3}\right| \geq 1$$

$\therefore$ Divergent

② $$\sum_{n=0}^{10} \left(\frac{3}{2}\right)^n \quad r = \frac{3}{2} \text{ and finite so...}$$

· converges to $\dfrac{1-\left(\frac{3}{2}\right)^{11}}{1-\frac{3}{2}} = \boxed{2\left(\frac{3}{2}^{11} - 1\right)}$

③ $$\sum_{n=1}^{\infty} 7(0.8)^{n-1} \quad \begin{matrix} a = 7 \\ r = .8 \end{matrix} \quad |.8| < 1 \therefore \text{convergent}$$

converges to $\dfrac{a}{1-r}$

so... $\dfrac{7}{1-.8} = \dfrac{7}{.2} = \boxed{35}$

for more... See
Section 8.2
Page 567

8.2 not geometric

Eric Straka

21. $\sum_{k=2}^{\infty} \frac{k^2}{k^2-1}$

$$\lim_{k\to\infty} \frac{k^2}{k^2-1} = \lim_{k\to\infty} \frac{1}{1-\frac{1}{k^2}} = 1$$

$\sum_{k=2}^{\infty} \frac{k^2}{k^2-1}$ diverges by nth term test for divergence

36. $.\overline{73} = .737373\ldots$

$$= \frac{73}{10^2} + \frac{73}{10^4} + \frac{73}{10^6} + \frac{73}{10^8} + \ldots$$

$$= \sum_{n=0}^{\infty} \frac{73}{10^{2n+2}} \qquad a = \frac{73}{100} \qquad r = \frac{1}{100}$$

$$= \frac{\frac{73}{100}}{1-\frac{1}{100}} = \frac{\frac{73}{100}}{\frac{99}{100}} = \frac{73}{99}$$

Sarah Kapp

8.3

1) let $f(n) = \frac{1}{n^5}$

$\int_1^\infty \frac{1}{n^5}$ is convergent by p-series test

since $p = 5 > 0$.

So, since $\int_1^\infty \frac{1}{n^5}$ is convergent,

$\sum_{n=0}^{\infty} \frac{1}{n^5}$ is convergent.

2). $\sum_{n=1}^{\infty} \frac{n^2-5n}{n^3+n+1}$

let $a_n = \frac{n^2-5n}{n^3+n+1}$ & let $b_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2-5n}{n^3+n+1} \cdot n = \lim_{n\to\infty} \frac{n^3-5n^2}{n^3+n+1} = 1$

$\sum_{n=1}^{\infty} b_n$ diverges by p-series test since $p=1$.

so $\sum_{n=1}^{\infty} a_n$ diverges as well.

so $\sum_{n=1}^{\infty} \frac{n^2-5n}{n^3+n+1}$ diverges by limit comparison test.

8.4- Answers

Dante Enriquez

29.) $\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$

Ratio test

$$\lim_{n\to\infty}\left|\frac{10^{n+1}}{(n+2)4^{2n+3}}\cdot\frac{(n+1)4^{2n+1}}{10^n}\right| = \lim_{n\to\infty}\left|\frac{10^{n+1}}{10^n}\cdot\frac{(n+1)}{(n+2)}\cdot\frac{4^{2n+1}}{4^{2n+3}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{10}{16}\cdot\frac{(n+1)}{(n+2)}\right| \quad \frac{5}{8}\lim_{n\to\infty}\left|\frac{(n+1)}{(n+2)}\right| = \frac{5}{8}\cdot 1 < 1$$

As the $\lim_{n\to\infty} = \frac{5}{8}$, which is less than 1, the series $\sum_{n=1}^{\infty}\frac{10^n}{(n+1)4^{2n+1}}$ is convergent.

8.4 - Answers

Dante Enriquez

7.) $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n-1}$

Use alt. series test.

1. Is $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n-1}$ a decreasing function?

Well

Let $f(x) = \cancel{\sum} (-1)^x \frac{3x-1}{2x-1}$

$$(-1)^{x+1} \frac{3(x+1)-1}{2(x+1)-1} = (-1)^{x+1} \frac{3x+2}{2x+1}$$

$$= (-1)^{x+1} \frac{3x+2}{2x+1} \geq (-1)^x \frac{3x+1}{2x-1}$$

This is an increasing function, therefore by the alt. series test $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n-1}$ is divergent.

ALTERNATING SERIES

8.4

MICHAEL POWER

SOLUTIONS

1) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

(i) LET $b_n = \frac{1}{2n+1}$

$$\frac{1}{2(n+1)+1} = \frac{1}{2n+3} < \frac{1}{2n+1}$$

SO THE FUNCTION IS DECREASING

(ii) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{2n+1} = 0$

SO THE SERIES CONVERGES BY THE ALTERNATING SERIES TEST

2) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+9}$

LET $b_n = \frac{n}{n^2+9}$ LET $f(x) = \frac{x}{x^2+9}$

(i) $\frac{x}{x^2+9} dx = \frac{(x^2+9) - x \cdot (2x)}{(x^2+9)^2}$

$$= \frac{(x^2+9) - 2x^2}{(x^2+9)^2}$$

$$= \frac{-x^2+9}{(x^2+9)^2}$$

$$= -\frac{1}{x^2+9} < 0$$

SO THE FUNCTION IS DECREASING

(ii) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n}{n^2+9}$

$$= 0$$

THE SERIES $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+9}$ CONVERGES BY THE ALTERNATING SERIES TEST

Sten Golds

Alternating Series (and Estimation

1.) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{10^n}$

$n_1 = \frac{1}{10}$

$n_2 = -\frac{1}{25}$

$n_3 = \frac{9}{1000}$

$n_4 = -\frac{1}{625}$

$n_5 = \frac{1}{4000}$

$n_6 = -\frac{9}{250000}$

Check if $|n_x| < \frac{1}{10000} = .0001$,

$|n_5| = .00025 \not< .0001$,

$|n_6| = .000036 < .0001$, Yay!

So to do the summation accurate to four decimal places use terms $n_1 - n_5$,

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{10^n} = \frac{1}{10} - \frac{1}{25} + \frac{9}{1000} - \frac{1}{625} + \frac{1}{4000} = \frac{1353}{20000} = \boxed{.0676}$$

2.) $$\frac{1}{1 \cdot 2^1} - \frac{1}{3 \cdot 2^3} + \frac{1}{5 \cdot 2^5} - \frac{1}{7 \cdot 2^7} \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)2^{2n+1}} = \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{1}{2}\right)^{2n+1}}{2n+1} = \boxed{\tan^{-1}\left(\frac{1}{2}\right)}$$

8.5 Power Series Solution

Jenny
Serluco

1. $$\sum_{n=1}^{\infty} n!(2x-1)^n$$

ratio test:

$$\lim_{n\to\infty} \left| (n+1)!(2x-1)^{n+1} \cdot \frac{1}{n!(2x-1)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)\cancel{!}(2x-1)^{n\cancel{+1}}}{\cancel{n!}\,\cancel{(2x-1)^n}} \right|$$

$$= \lim_{n\to\infty} n+1 \cdot (2x-1) = (2x-1) \lim_{n\to\infty} n+1 = \infty$$

Interval: $\{\frac{1}{2}\}$

Radius: 0

8.5 Power Series Solution

Jenny Serluco

2. $J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!\,2^{2n+1}}$

ratio test: $\lim_{n \to \infty} \left| \frac{(1)^{n+1} x^{2n+3}}{(n+1)!(n+2)!\,2^{2n+3}} \cdot \frac{n!(n+1)!\,2^{2n+1}}{1^n x^{2n+1}} \right|$

$\frac{x^2}{4(n+2)(n+1)} = x^2 \lim_{n \to \infty} \frac{1}{4(n+2)(n+1)} = 0$

$(-\infty, \infty)$

8.6: 3, 13

Almonther Alshareef

1) $f(x) = \frac{1}{1+x}$

we know that $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

$= \sum_{n=0}^{\infty} x^n, |x| < 1 \cdots (1)$

Replace x by -x in (1)

$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n$

$\sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \cdots$

It's a geometric ~~function~~ series, converges when magnitude of ratio less than one:

$|-x| < 1$

$|x| < 1 \Rightarrow$ $\boxed{\text{Interval of convergence is } (-1, 1)}$

2) $f(x) = \ln(5-x)$

$\frac{1}{5-x} = \frac{1}{5}\left(\frac{1}{(1-\frac{x}{5})}\right) = \frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{x}{5}\right)^n \Rightarrow \frac{1}{5-x} = \frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{x}{5}\right)^n$

$\Rightarrow \frac{\ln(5-x)}{-1} = \sum_{n=0}^{\infty} \frac{1}{5^{n+1}} \cdot \frac{x^{n+1}}{n+1}$

$\Rightarrow \ln(5-x)$ converges when $\left|\frac{x}{5}\right| < 1$

$\boxed{\text{Radius of convergence} = 5}$

1. $f(x) = x\cos(x)$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \cdots$$

$$x\cos(x) = x\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n)!}$$

2. $f(x) = xe^{-x}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$xe^x = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$$

$$xe^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n!}$$

Justin Malcum
Section 5
Making new
Maclaurin
Series

8.7

Constructing Maclaurin Series
Holly Fuhrer
(solutions)

1. $f(x) = (1-x)^{-2}$

$f'(x) = -2 \cdot -1(1-x)^{-3}$

$f''(x) = -3 \cdot 2 \cdot -1(1-x)^{-4}$

$f'''(x) = -4 \cdot 3 \cdot 2 \cdot -1(1-x)^{-5}$

$f(0) = 1$

$f'(0) = 2!$

$f''(0) = 3!$

$f'''(0) = 4!$

$$f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$= 1 + \frac{2}{1!}x + \frac{3!}{2!}x^2 + \frac{4!}{3!}x^3 + \dots$$

$$= \sum_{n=0}^{\infty} \frac{(n+1)!}{(n)!}x^n$$

$$= \sum_{n=0}^{\infty} (n+1)x^n$$

2. $f(x) = e^{5x}$

$f'(x) = 5e^{5x}$

$f''(x) = 5^2 e^{5x}$

$f'''(x) = 5^3 e^{5x}$

$f(0) = 1$

$f'(0) = 5$

$f''(0) = 5^2$

$f'''(0) = 5^3$

$$f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$= 1 + \frac{5}{1!}x + \frac{5^2}{2!}x^2 + \frac{5^3}{3!}x^3 + \dots$$

$$= \sum_{n=0}^{\infty} \frac{5^n}{n!}x^n$$

① a) $f(x) = \cos(2x) - 1 + 2x^2$ 8.7

$$\cos(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}(-1)^n = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$\cos(2x) = \sum_{n=0}^{\infty} \frac{(2x)^{2n}}{(2n)!}(-1)^n = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \ldots$$

$$= 1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 \ldots$$

Therefore the first two non-zero terms are:

$$f(x) = \cos(2x) - 1 + 2x^2$$

$$f(x) = \left(1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \ldots\right) - 1 + 2x^2$$

$$\boxed{f(x) = \frac{2}{3}x^4 - \frac{4}{45}x^6 + \ldots}$$

b) using the expansion from part a):

$$\lim_{x \to 0} \frac{\cos(2x) - 1 + 2x^2}{x^4} = \lim_{x \to 0} \frac{\frac{2}{3}x^4 - \frac{4}{45}x^6 + \ldots}{x^4}$$

$$= \lim_{x \to 0} \left(\frac{2}{3} - \cancel{\frac{4}{45}x^2}^{\,0} + \cancel{\ldots}^{\,0}\right)$$

Therefore

$$\boxed{\lim_{x \to 0} \frac{\cos(2x) - 1 + 2x^2}{x^4} = \frac{2}{3}}$$

Sec. 3
Quinn Cantrell

② $f(x) = \sin(3x^2)$ 8.7

$$\sin(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} \cdot (-1)^n = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\sin(3x^2) = \sum_{n=0}^{\infty} \frac{(3x^2)^{2n+1}}{(2n+1)!} = (3x^2) - \frac{(3x^2)^3}{3!} + \frac{(3x^2)^5}{5!} - \dots$$

$$= 3x^2 - \frac{3^3 x^6}{3!} + \frac{3^5 x^{10}}{5!} - \frac{3^7 x^{14}}{7!} + \dots$$

We know from the MacLaurin series formula;

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

that the coefficent for x^{14} will be

$$\frac{f^{(14)}(0)}{14!}$$

Using the power series of $\sin(3x^2)$ above;

$$\frac{f^{(14)}(0)}{14!} = \frac{-3^7}{7!}$$

Thus,

$$\boxed{f^{14}(0) = \frac{14!\, 3^7}{7!}}$$

Quinn Cantrell Sec3

Berea Bearyman
Review Problems
12/11/13

Section 8.8 Answers

(13) a)

$f(x) = x^{2/3}$ $\quad a = 1 \quad n = 3 \quad 0.8 \leq x \leq 1.2$

$f(x) = x^{2/3} = f(1) = 1 \qquad f'(x) = 2/3 x^{-1/3} = f'(1) = 2/3$

$f''(x) = -\frac{2}{9}x^{-4/3} = f''(1) = -2/9 \qquad f'''(x) = \frac{8}{27}x^{-7/3} = 8/27$

$$T_n(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$T_3(x) = \sum_{n=0}^{\infty} f(x) + \frac{f'(x)}{1!}(x-1) + \frac{f''(x)}{2!}(x-1)^2 + \frac{f'''(x)}{3!}(x-1)^3$$

$$T_3(x) = \sum_{n=0}^{\infty} 1 + \frac{2/3}{1!}(x-1) + \frac{(-2/9)}{2!}(x-1)^2 + \frac{8/27}{3!}(x-1)^3$$

$$T_3(x) = \sum_{n=0}^{\infty} 1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{4}{81}(x-1)^3$$

b)

$f(x) \approx T_3(x)$

$$x^{2/3}\Big]_{0.8}^{1.2} \approx 1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{4}{81}(x-1)^3 \Big]_{0.8}^{1.2}$$

$$(1.2)^{2/3} - (0.8)^{2/3} \approx 1 + \frac{2}{3}(1.2-1) - \frac{1}{9}(1.2-1)^2 + \frac{4}{81}(1.2-1)^3 - \left(1 + \frac{2}{3}(0.8-1) - \frac{1}{9}(0.8-1)^2 + \frac{4}{81}(0.8-1\right.$$

$0.2674693586 \approx 1.129283951 - 0.8618271605$

$0.2674693586 \approx 0.2674567905$

$|R_3(x)| = |f(x) - T_3(x)|$

$|R_3(x)| = 0.2674693586 - 0.2674567905$

$|R_3(x)| = 0.00001$