Josh Rementeria

Section 5.2 Review SOLUTIONS

18) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos x_i}{x_i} \Delta x$, $[\pi, 2\pi]$

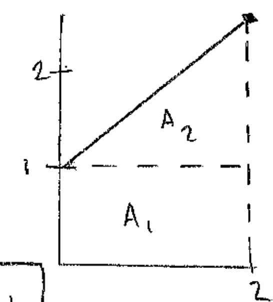
In general,

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \text{ on interval } [a,b]$$

$$\therefore \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos x_i}{x_i} \Delta x \text{ on } [\pi, 2\pi] = \boxed{\int_{\pi}^{2\pi} \frac{\cos x}{x}\,dx}$$

31) a) $\int_0^2 f(x)\,dx$

(for a-d, refer to graph on question page of textbook page 354)



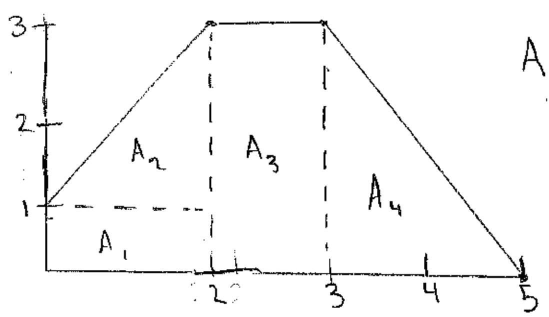
A_1 = Rectangle
$= l \cdot w$
$= 2 \cdot 1 = 2 = A_1$

A_2 = Triangle
$= \frac{1}{2}bh$
$= \frac{1}{2}(2)(2) = 2 = A_2$

$\boxed{\int_0^2 f(x)\,dx = 4}$

$A = \underline{\underline{A_1 + A_2}} = 2 + 2 = \underline{\underline{4}} = A$

b) $\int_0^5 f(x)\,dx$



$\boxed{\int_0^5 f(x) = 10}$

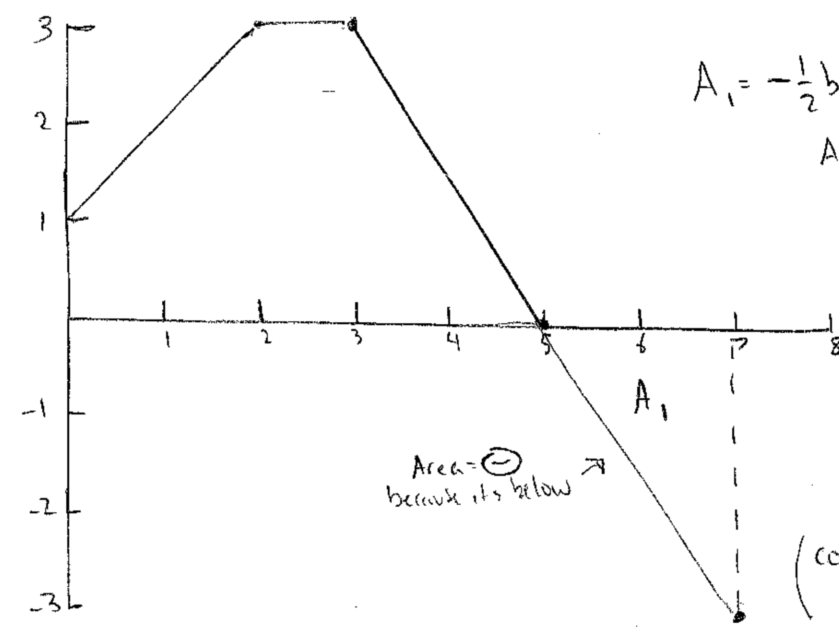
$A = \underbrace{A_1 + A_2}_{\text{see part (a)}} + A_3 + A_4$

$A_3 = l \cdot w$
$= 3 \cdot 1$
$= 3 = A_3$

$A_4 = \frac{1}{2}bh$
$= \frac{1}{2}(2)(3)$
$= 3 = A_4$

$A = A_1 + A_2 + A_3 + A_4 = 4 + 3 + 3 = \underline{\underline{10}}$

c) $\int_5^7 f(x)\,dx$



$A_1 = -\frac{1}{2}bh = -\frac{1}{2}(2)(3)$

$A_1 = -3$

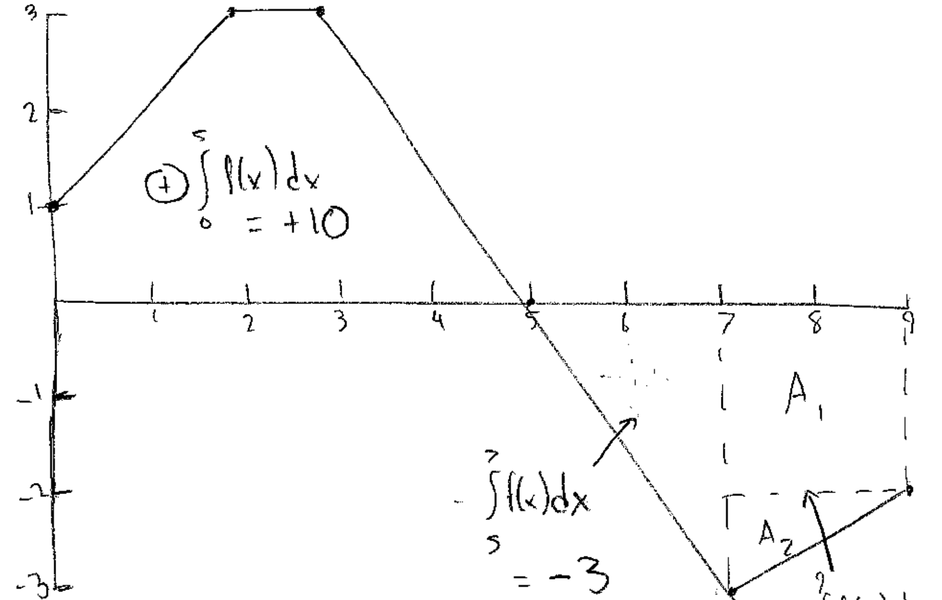
$\boxed{\int_5^7 f(x)\,dx = -3}$

(continued on next page) ⟹

Josh Rementeria

Section 5.2 Review SOLUTIONS (continued)

31) d) $\int_0^9 f(x)\,dx =$



$-\int_7^9 f(x)\,dx$

$= A_1 + A_2$

$= s^2 + \frac{1}{2}bh$

$= 4 + \frac{1}{2}(1)(2) = 5$

$\int_7^9 f(x)\,dx = \ominus 5$ (Below x-axis)

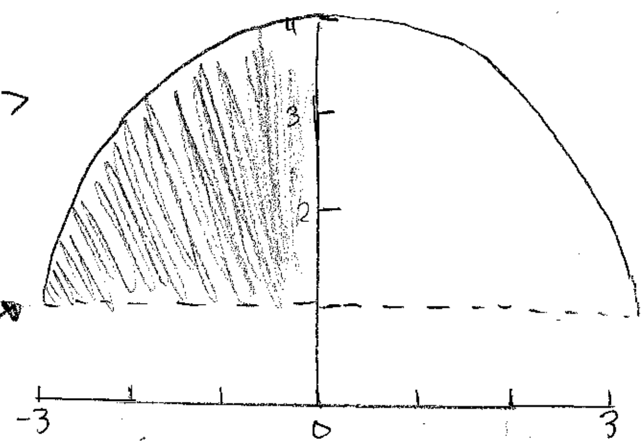
$\int_0^9 f(x)\,dx = \int_0^7 f(x)\,dx + \int_7^9 f(x)\,dx$

$= 10 - 3 - 5$

$\boxed{\int_0^9 f(x)\,dx = 2}$

35) $\int_{-3}^{0} \left(1 + \underbrace{\sqrt{9-x^2}}\right) dx \Rightarrow$

semicircle w/ r = 3



Area of Semicircle = $\frac{\pi r^2}{2}$ Area of $\frac{1}{2}$ Semicircle = $\frac{\pi r^2}{4}$

$\int_{-3}^{0} \left(1 + \sqrt{9-x^2}\right) dx \Rightarrow \frac{\pi (3)^2}{4} = \boxed{\frac{9\pi}{4} = \int_{-3}^{0}\left(1+\sqrt{9-x^2}\right)dx}$

Simon Truong

5.3 Evaluating Definite Integrals
(Answer Key)

29) $\int_{-1}^{2} (x - 2|x|)\,dx$

$\int_{-1}^{0} (x - 2(-x))\,dx + \int_{0}^{2} (x - 2x)\,dx$

$\int_{-1}^{0} (3x)\,dx + \int_{0}^{2} (-x)\,dx$

$3\int_{-1}^{0} x\,dx - \int_{0}^{2} x\,dx$

$3\left[\frac{1}{2}x^2\right]_{-1}^{0} - \left[\frac{1}{2}x^2\right]_{0}^{2}$

$-\frac{3}{2} - 2$

$= \boxed{-\frac{7}{2}}$

59) $v(t) = 3t - 5, \quad 0 \le t \le 3$

a) $\int_{0}^{5/3} (3t-5)\,dt + \int_{5/3}^{3} (3t-5)\,dt$

$\left[\frac{3}{2}t^2 - 5t\right]_{0}^{5/3} + \left[\frac{3}{2}t^2 - 5t\right]_{5/3}^{3}$

$\left(\frac{25}{6} - \frac{25}{3}\right) + \left[\left(\frac{27}{2} - 15\right) - \left(\frac{25}{6} - \frac{25}{3}\right)\right]$

$-\frac{25}{6} - \frac{3}{2} + \frac{25}{6} = \boxed{-\frac{3}{2}\,m}$

b) $\int_{0}^{5/3} |3t-5|\,dt + \int_{5/3}^{3} (3t-5)\,dt$

$\int_{0}^{5/3} (-3t+5)\,dt + \int_{5/3}^{3} (3t-5)\,dt$

$\left[-\frac{3}{2}t^2 + 5t\right]_{0}^{5/3} + \left[\frac{3}{2}t^2 - 5t\right]_{5/3}^{3}$

$\left(-\frac{25}{6} + \frac{25}{3}\right) + \left(-\frac{3}{2} + \frac{25}{6}\right)$

$\boxed{\frac{41}{6}\,m}$

67) $C(x)$ is cost of producing x yards.

$\int_{2000}^{4000} C'(x)$ is the increase in cost.

$\int_{2000}^{4000} (3 - 0.01x + 0.000006x^2)\,dx$

$= \left[3x - \frac{0.01x^2}{2} + \frac{0.000006}{3}x^3\right]_{2000}^{4000}$

$= 60{,}000 - 2000$

$= \boxed{\$58{,}000}$

Julia Norman Section 5.4 Fundamental Theorem of Calculus

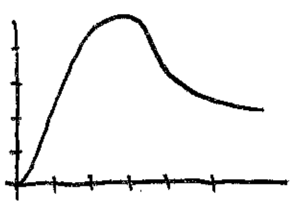
Answers

1. a. $g(0) = 0$
$g(1) = 2$
$g(2) = 5$
$g(3) = 7$
$g(6) = 3$

these values were found by adding up the areas of the boxes under the graph of f

b. g is increasing on $(0,3)$ because this is the interval where the graph of f is positive.

c. g has a ~~value~~ maximum value at $x=3$, because ~~on~~ the graph of y crosses from positive to negative at $x=3$.

d. 

2. $$\int_1^x \frac{1}{t^3+1}\,dt = F(x) \qquad F'(x) = \frac{d}{dx}\int_1^{x} \boxed{\frac{1}{t^3+1}}\,dt = \boxed{\frac{1}{x^3+1}}$$

3. $$\int_x^{\pi} \sqrt{1+\sec t}\,dt = F(x) = -\int_{\pi}^{x} \boxed{\sqrt{1+\sec t}}\,dt$$

$$\boxed{g'(x) = -\sqrt{1+\sec x}}$$

Michael Sloan

Chapter 5.5 The Substitution Rule
(Answer #3)

Integrating Even and Odd Functions

* If f is even $(f(-x)=f(x))$ then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$
* If f is odd $(f(-x)=-f(x))$ then $\int_{-a}^{a} f(x)\,dx = 0$

a $\int_{-9}^{9} \frac{\tan(x)}{1+x^2+x^4}\,dx$

$$f(-x) = \frac{\tan(-x)}{1+(-x)^2+(-x)^4} = -\frac{\tan(x)}{1+x^2+x^4} = -f(x)$$

so $\frac{\tan(x)}{1+x^2+x^4}$ is odd

Thus $\int_{-9}^{9} \frac{\tan(x)}{1+x^2+x^4}\,dx = 0$

b $\int_{-2}^{2} (x^6+1)\,dx$

$$f(-x) = (-x)^6+1 = x^6+1 = f(x)$$

So x^6+1 is even

Thus we have: $2\int_{0}^{2} x^6+1\,dx$

$$\left[2\left(\tfrac{1}{7}x^7+x\right)\right]_0^2$$

$$2\left(\frac{2^7}{7}+2-0\right) = \boxed{40.571}$$

Michael Sloan

Chapter 5.5 The Substitution Rule

(Answer #1)

U Substitution

1. $\int x^3 \cos(x^4+2)\,dx$

Let $u = x^4 + 2$

$du = 4x^3\,dx$

so

$dx = \frac{1}{4x^3}\,du$

$\int x^3 \cos(u) \frac{1}{4x^3}\,du$

$\int \frac{1}{4} \cos(u)\,du$

$\frac{1}{4} \int \cos(u)\,du$

$\frac{1}{4}\left(-\sin(u)\right) + C$

$-\frac{1}{4} \sin(u) + C$

Substitute u back into the equation

$= -\frac{1}{4} \sin(x^4+2) + C$

Michael Sloan

Chapter 5.5 The Substitution Rule
(Answer #2)

Integrating Definite Integrals

2. $$\int_{1/6}^{1/2} \csc(\pi t)\cot(\pi t)\,dt$$

$$\int_{1/6}^{1/2} \frac{1}{\sin(\pi t)} \cdot \frac{\cos(\pi t)}{\sin(\pi t)}\,dt$$

$$\int_{1/6}^{1/2} \frac{\cos(\pi t)}{\sin^2(\pi t)}\,dt$$

$$u = \sin(\pi t)$$
$$du = \pi\cos(\pi t)\,dt$$
$$dt = \frac{1}{\pi\cos(\pi t)}\,du$$

$$\frac{1}{\pi}\int_{1/2}^{1} \frac{1}{u^2}\,du$$

$$u = \sin\left(\pi\left(\tfrac{1}{6}\right)\right) = \sin\left(\tfrac{\pi}{6}\right) = \frac{1}{2}$$

$$\frac{1}{\pi}\int_{1/2}^{1} u^{-2}\,du$$

$$u = \sin\left(\pi \cdot \tfrac{1}{2}\right) = \sin\left(\tfrac{\pi}{2}\right) = 1$$

$$\left[\frac{1}{\pi}\left(-\frac{1}{u}\right) + C\right]_{1/2}^{1}$$

$$\left[-\frac{1}{\pi u} + C\right]_{1/2}^{1}$$

$$-\frac{1}{\pi(1)} - \left(-\frac{1}{\frac{\pi}{2}}\right)$$

$$-\frac{1}{\pi} + \frac{2}{\pi} = \boxed{\frac{1}{\pi}}$$

Answer Sheet

Rakan Alhajeri

5.6

11) $\int \arctan 4t \, dt = \int \tan^{-1}(4t)\,dt$

$u = \arctan(4t)$, $dv = dt$

$du = \dfrac{1}{1+(4t)^2} \cdot 4dt = \dfrac{4}{1+16t^2}\,dt$, $v = t$

$$\int \arctan 4t \, dt = \arctan(4t)\,t - \int \frac{4t}{1+16t^2}\,dt$$

$$= t\arctan(4t) - 4\int \frac{t}{1+16t^2}\,dt$$

$1 + 16t^2 = \theta$

$32\,t\,dt = d\theta$

$t\,dt = \dfrac{d\theta}{32}$

$$= t\arctan(4t) - 4\int \frac{1}{\theta} \cdot \frac{d\theta}{32}$$

$$= t\arctan(4t) - \frac{1}{8}\int \frac{1}{\theta}\,d\theta$$

$$= t\arctan(4t) - \frac{1}{8}\ln|\theta| + C$$

$$= t\arctan(4t) - \frac{1}{8}\ln|1+16t^2| + C$$

23) $\int_1^2 (\ln x)^2\,dx$

$\int (\ln x)^2\,dx \qquad u = (\ln x)^2 \qquad dv = dx$

$\qquad du = 2(\ln x)\cdot\frac{1}{x}\,dx\ , \quad v = x$

$= (\ln x)^2 x - \int x \cdot 2(\ln x)\cdot\frac{1}{x}\,dx$

$= x(\ln x)^2 - 2\int \ln(x)\,dx$

$= x(\ln x)^2 - 2\left[\ln(x)x - \int \frac{1}{x}\cdot x\,dx\right]$

$= x(\ln x)^2 - 2\left[\ln(x)x - \int dx\right]$

$= x(\ln x)^2 - 2x\ln(x) + 2x + C$

$\int_1^2 (\ln x)^2 dx = \left[x(\ln x)^2 - 2x\ln(x) + 2x\right]_1^2$

$= \left[2(\ln 2)^2 - 4\ln(2) + 4 - (\ln 1)^2 + 2\ln 1 - 2\right]$

$= 2(\ln 2)^2 - 4\ln 2 + 2$

25. $\int \cos\sqrt{x}\,dx$

$w = \sqrt{x}$ $\quad dw = \frac{dx}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,dw$

$= 2w\,dw$

$= 2\int w\cos w\,dw$

$u = w$ $\quad dv = \cos w\,dw$

$du = dw$ $\quad v = \sin w\,dw$

$= 2(w\sin w - \int \sin w\,dw)$

$= 2(w\sin w + \cos w + C)$

$= 2(\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}) + C$

Sarina Haghighat 5.7 (trig integrals) Solutions

1) $$\int_0^{\pi/3} \frac{\sin x + \sin x \tan^2 x}{\sec^2 x}$$

$$\int_0^{\pi/3} \frac{\sin x (1 + \tan^2 x)}{\sec^2 x}$$

notice: $1 + \tan^2 x = \sec^2 x$

$$\int_0^{\pi/3} \frac{\sin x \cancel{(1 + \tan^2 x)}}{\cancel{\sec^2 x}}$$

$$\int_0^{\pi/3} \sin x$$

$$-\cos x \Big|_0^{\pi/3}$$

$$-\cos\left(\frac{\pi}{3}\right) - \left[-\cos(0)\right]$$

$$-\frac{1}{2} - (-1)$$

$$\boxed{\frac{1}{2}}$$

2.

$$\int \sin^3(x) \cos^2(x)\, dx$$

$$\int \sin^2(x) \cos^2(x) \sin(x)\, dx$$

rewrite: $\sin^2(x)$ as $(1-\cos^2(x))$

$$\int (1-\cos^2(x)) \cos^2(x) \sin(x)\, dx$$

let $u = \cos(x)$

$du = -\sin(x)\,dx$

$-du = \sin x\, dx$

$$-\int (1-u^2)(u^2)\, du$$

$$-\int u^2 - u^4\, du$$

$$-\left[\frac{u^3}{3} - \frac{u^5}{5} + C\right]$$

$$\frac{-u^3}{3} + \frac{u^5}{5} + C$$

$$\boxed{\frac{-\cos^3 x}{3} + \frac{\cos^5 x}{5} + C}$$

Sarina Haghighat 5.7 (trig integrals) Solutions

3) $\displaystyle\int_0^{\pi/6} \tan^2(x)\sec^4(x)\,dx$

$$\int_0^{\pi/6} \tan^2(x)\sec^2(x)\sec^2(x)\,dx$$

rewrite one $\sec^2(x)$ as : $1+\tan^2(x)$

$$\int_0^{\pi/6} \tan^2(x)\sec^2(x)\left(1+\tan^2(x)\right)dx$$

let $u = \tan(x)$
$du = \sec^2(x)\,dx$

Rewrite Bounds
when $x=0$ $u=0$
$x=\frac{\pi}{6}$ $u=\frac{\sqrt{3}}{3}$

$$\int_0^{\sqrt{3}/3} u^2(1+u^2)\,du$$

$$\int_0^{\sqrt{3}/3} u^2+u^4\,du$$

$$\frac{u^3}{3}+\frac{u^5}{5}\Bigg|_0^{\sqrt{3}/3}$$

$$\left[\frac{\left(\frac{\sqrt{3}}{3}\right)^3}{3}+\frac{\left(\frac{\sqrt{3}}{3}\right)^5}{5}\right]-\left[0+0\right]$$

$$\frac{3\sqrt{3}}{81}+\frac{9\sqrt{3}}{1215}$$

$$\frac{\sqrt{3}}{27}+\frac{\sqrt{3}}{135}\approx 0.07698$$

5.7 (Trig Subs)

Austin Hirsh

8) $\int \tan^5 x \sec^3 x \, dx$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\tan^2 x = \sec^2 \cancel{x} - 1$$

$$u = \sec x$$

$$du = \sec x \tan x$$

$= \int \tan x (\tan^2 x)^2 \sec x (\sec^2 x) dx$

$= \int \tan x \, (\sec^2 \cancel{x} - 1)^2 \sec x \sec^2 x \, dx$

$= \int (u^2-1)^2 u^2 du$

$= \int (u^2-1)(u^2-1) u^2 du$

$= \int (u^4 - 2u^2 + 1) u^2 du$

$= \int u^6 - 2u^2 + u^2 du$

$= \frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3 + c = \boxed{\frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + c}$

16) $\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}} dx \quad x = 4\sin\theta \quad dx = 4\cos\theta \, d\theta$

$= \int_0^{\pi/3} \frac{64\sin^3\theta \cdot 4\cos\theta \, d\theta}{\sqrt{16 - 16\sin^2\theta}} = \int_0^{\pi/3} \frac{64\sin^3\theta \cdot \cancel{4}\cancel{\cos\theta} \, d\theta}{\cancel{4}\cancel{\cos\theta}}$

$= \int_0^{\pi/3} 64\sin^3\theta \, d\theta = \int_0^{\pi/3} 64\sin^2\theta \sin\theta \, d\theta$

$= \int_0^{\pi/3} 64(1-\cos^2\theta)(\sin\theta) \, d\theta$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$x = \frac{\pi}{3} \to u = \frac{1}{2}$

$x = 0 \to u = 1$

$= 64\int_{1/2}^{1} (1-u^2) \, du$

$= 64\left[u - \frac{1}{3}u^3\right]_{1/2}^{1}$

$= 64\left(1 - \frac{1}{3}\right) - 64\left(\frac{1}{2} - \frac{1}{3}\left(\frac{1}{2}\right)^3\right)$

$= 64\left(\frac{2}{3}\right) - 64\left(\frac{11}{24}\right)$

$= \frac{128}{3} - \frac{88}{3}$

$= \frac{40}{3}$

Continue to Second Page

5.7 (Trig Subs) (Page 2) Austin Hirsh

18) $\int \frac{x^3}{\sqrt{x^2+1}}$ $\quad x = \tan\theta \quad\quad dx = \sec^2\theta\, d\theta$

$= \frac{(\tan\theta)^3}{\sqrt{(\tan^2\theta+1}} \cdot \sec^2\theta\, d\theta$

$= \int \frac{\tan^3\theta}{\cancel{\sec\theta}} \cdot \sec^{\cancel{2}}\theta\, d\theta$

$= \int \tan^2\theta \cdot \tan\theta \cdot \sec\theta\, d\theta$

$= \int (\sec^2\theta - 1)\tan\theta \sec\theta\, d\theta$

$u = \sec\theta$

$du = \sec\theta \tan\theta\, d\theta$

$= \int (u^2-1)\, du$

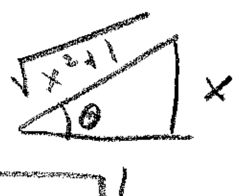
$= \frac{1}{3}u^3 - u + C$

$= \frac{1}{3}\sec^3\theta - \sec\theta + C$

$= \frac{1}{3}\left(\sqrt{x^2+1}\right)^3 - \sqrt{x^2+1} + C$

$x = \tan\theta$

$x = \frac{O}{A} \quad\quad \frac{1}{\cos} = \frac{H}{A}$



$\sec\theta = \sqrt{x^2+1}$

$= \boxed{\frac{1}{3}(x^2+1)\sqrt{x^2+1} - \sqrt{x^2+1} + C}$

or

$\boxed{\sqrt{x^2+1}\left(\frac{1}{2}(x^2+1) - 1\right) + C}$

Joe Taylor

Section 5.7 (Partial Fractions)

$$\int \frac{x^2-29x+5}{(x-4)^2(x^2+3)}\,dx$$

$$\frac{x^2-29x+5}{(x-4)^2(x^2+3)} = \frac{A}{(x-4)} + \frac{B}{(x-4)^2} + \frac{Cx+D}{(x^2+3)}$$

$$x^2-29x+5 = A(x-4)(x^2+3) + B(x^2+3) + (Cx+D)(x-4)^2$$

* multiply out

$$= Ax^3 - 4A^2 + 3Ax - 12A + Cx^3 - 8Cx^2 + 16Cx + Bx^2$$
$$+ 3B + Dx^2 - 8Dx + 16D$$

* collect like terms

$$= (A+C)x^3 + (-4A+B-8C+D)x^2 + (3A+16C-8D)x - 12A + 3B + 16D$$

* equate coefficient to other side $(x^2-29x+5)$

$(A+C)=0 \quad -4A+B-8C+D=1 \quad 3A+16C-8D=-29$

$-12A+3B+16D=5$

$\therefore \quad A=1 \quad B=-5 \quad C=-1 \quad D=2$

$$= \int \frac{1}{x-4} - \frac{5}{(x-4)^2} + \frac{-x+2}{x^2+3}\,dx$$

$$= \int \frac{1}{x-4}\,dx - \int \frac{5}{(x-4)^2}\,dx - \int \frac{x}{x^2+3}\,dx + \int \frac{2}{x^2+3}\,dx$$

$$= \ln|x-4| + \frac{5}{x-4} - \frac{1}{2}\ln|x^2+3| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

Joe Taylor

Section 5.7 Partial Fractions

$$\int \frac{5x-4}{2x^2+x-1}\,dx$$

$$\frac{5x-4}{2x^2+x-1} = \frac{5x-4}{(x+1)(2x-1)}$$

$$\frac{5x-4}{(x+1)(2x-1)} = \frac{A}{x+1} + \frac{B}{2x-1} \longrightarrow 5x-4 = A(2x-1) + B(x+1)$$

set $x=-1$: $5(-1)-4 = A(2(-1)-1) + B(0)$

$$-9 = A(-3)$$
$$3 = A$$

set $x=\frac{1}{2}$: $5(\frac{1}{2})-4 = A(0) + B(\frac{1}{2}+1)$

$$-1.5 = B(1.5)$$
$$-1 = B$$

$$= \int \frac{3}{x+1} - \frac{1}{2x-1}\,dx = 3\int \frac{1}{x+1} - \int \frac{1}{2x-1}$$

$\ln(u) = \frac{1}{u}$ $u = x+1$ $u = 2x-1$

$$= 3\ln|x+1| - \frac{1}{2}\ln|2x-1| + C$$

Joe Taylor
Section 5.7 (partial fractions)

$$\int \frac{2x^2 - x + 4}{x^3 + 4x}\,dx$$

* $x^3 + 4x = x(x^2+4)$ and can't be factored further so,

$$\frac{2x^2 - x + 4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx + C}{x^2+4}$$

$$2x^2 - x + 4 = A(x^2+4) + (Bx+c)(x)$$

$$= (A+B)x^2 + Cx + 4A$$

* Equate coefficients to the other side

$2 = A + B \qquad C = -1 \qquad 4A = 4 \rightarrow A = 1$

$\therefore\ A = 1 \qquad B = 1 \qquad C = -1$

$$= \int \frac{1}{x} + \frac{x-1}{x^2+4}\,dx$$ * split up second part to get:

$$= \int \frac{1}{x}\,dx + \int \frac{x}{x^2+4}\,dx - \int \frac{1}{x^2+4}\,dx$$

* knowing/given $\int \frac{1\ dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$ then:

$$= \ln|x| + \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$$

Mike Gross 5.9

5. a) $\int_0^2 \frac{x}{1+x^2}\,dx \approx \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4)\right.$

$n=10$

$\left. + f(\bar{x}_5) + f(\bar{x}_6) + f(\bar{x}_7) + f(\bar{x}_8) + f(\bar{x}_9)\right.$

$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) \quad \Delta x = \frac{b-a}{n}$

$M_{10} \approx .806598$

b) $\int_0^2 \frac{x}{1+x^2}\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4)\right.$

$n=10$

$+ 4f(x_5) + 2f(x_6) + 4f(x_7) + 2f(x_8)$

$\left. + 4f(x_9) + f(x_{10})\right]$

$\Delta x = \frac{(b-a)}{n}$

$S_{10} \approx .804779$

c) $\int_0^2 \frac{x}{1+x^2}\,dx = \frac{\ln(x^2+1)}{2}\Big|_0^2 \quad \frac{\ln(5)}{2} - \frac{\ln(1)}{5} = \frac{\ln(5)}{2}$

$= .804718956\cancel{2}$

$.8047189567 - .806598 = E_{M10} = -.001879$

$.8047189567 - .804779 = E_{S10} = -.0000600438$

7

Mike Gow 5.9

a) $\int_0^2 \sqrt[4]{1+x^2}\,dx \approx \frac{\Delta x}{2}\Big[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3)$

$n = 8$

$+ 2f(x_4) + 2f(x_5) + 2f(x_6) + 2f(x_7)$

$+ 2f(x_8) + 2f(x_9) + f(x_{10}) \Big]$

$T_8 \approx 2.413790$ $\Delta x = \frac{(b-a)}{n}$ $x_i = a + i\Delta x$

b) $\int_0^2 \sqrt[4]{1+x^2}\,dx \approx \Delta x \Big[f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4) +$

$f(\bar{x}_5) + f(\bar{x}_6) + f(\bar{x}_7) + f(\bar{x}_8) \Big]$

$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$ $\Delta x = \frac{b-a}{n}$

$M_8 \approx 2.411453$

c) $\int_0^2 \sqrt[4]{1+x^2}\,dx \approx \frac{\Delta x}{3}\Big[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3)$

$+ 2f(x_4) + 4f(x_5) + 2f(x_6)$

$+ 4f(x_7) + 2f(x_8) + 4f(x_9)$

$+ f(x_{10}) \Big]$

$\Delta x = \frac{(b-a)}{n}$

$S_{10} \approx 2.412232$

1a

Mike [illegible] 5.9

a) $\int_0^{\pi} \sin(x)dx \approx T_{10} \approx \frac{\Delta x}{2}\Big[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + 2(x_5)$
$+2f(x_6) + 2f(x_7) + 2f(x_8) + 2f(x_9) + f(x_{10})\Big]$

$\Delta x = \left(\frac{b-a}{10}\right) \quad x_i = a + i\Delta x$

$\approx M_{10} \approx \Delta x\Big[f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4) + f(\bar{x}_5) + f(\bar{x}_6)$
$+ f(\bar{x}_7) + f(\bar{x}_8) + f(\bar{x}_9) + f(\bar{x}_{10})\Big]$

$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) \quad \Delta x = \frac{b-a}{n}$

$\approx S_{10} \approx \frac{\Delta x}{3}\Big[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4)$
$+ 4f(x_5) + 2f(x_6) + 4f(x_7) + 2f(x_8)$
$+ 4f(x_9) + f(x_{10})\Big]$

$\Delta x = \left(\frac{b-a}{n}\right)$

$T_{10} \approx 1.983524 \quad M_{10} \approx 2.008248 \quad S_{10} \approx 2.000110$

b) $\int_0^{\pi} \sin(x)dx \quad -\cos(x)\Big|_0^{\pi} \quad -\cos(\pi) + \cos(0) = 2$

$2 - 1.983524 = E_{T_{10}} = .016476$

$2 - 2.008248 = E_{M_{10}} = -.008248$

$2 - 2.000110 = E_{S_{10}} = -.000110$

① $$\int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}}\,dw$$

$$= \lim_{A\to-\infty} \int_A^{-1} \frac{1}{\sqrt{2-w}}\,dw$$

$$= -2 \lim_{A\to-\infty} (2-w)^{1/2}\Big|_A^{-1} = -2\left[\sqrt{2-(-1)} - \sqrt{2-A}\right]$$

$$= -2\left[\sqrt{3} - \sqrt{2-A}\right] = \infty$$

$\boxed{\text{Divergent}}$

② $$\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$$

$$= \lim_{A\to0^+} \int_A^1 \frac{\ln x}{\sqrt{x}}\,dx$$

$u = \ln x \quad dv = \frac{1}{\sqrt{x}}\,dx$

$du = \frac{1}{x}\,dx \quad v = 2x^{1/2}$

$$2\sqrt{x}\ln x - \int \frac{2\sqrt{x}}{x}\,dx$$

$$= 2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}\,dx$$

$$= 2\sqrt{x}\ln x - 2(2\sqrt{x})$$

$$\lim_{A\to0^+} 2\sqrt{x}(\ln x - 2)\Big|_A^1 =$$

$$= 0 - 4 - (2\sqrt{A}(\ln A - 2))$$

$$= -4 - \lim_{A\to0^+}\left[2\sqrt{A}(\ln A - 2)\right]$$

$$= -4 - \lim_{A\to0^+} \frac{\ln A - 2}{\frac{1}{2\sqrt{A}}} \quad \frac{\infty}{\infty} \Rightarrow \frac{\frac{1}{A}}{-\frac{1}{4}A^{-3/2}}$$

$$= \frac{1}{A} \div \frac{-1A^{3/2}}{4} = \frac{1}{A} \times -4A^{3/2}$$

$$= -4A^{1/2} = -4\sqrt{A}$$

$$= -4 - \lim_{A\to0^+} \underbrace{-4\sqrt{A}}_{0} = -4$$

$\boxed{\text{Converges to } -4}$

Maureen Cobile Sec 5.10 Answers (1)

(3) $\int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx$

$$\frac{x}{\sqrt{x^4}} = \frac{x}{x^2} = \frac{1}{x}$$

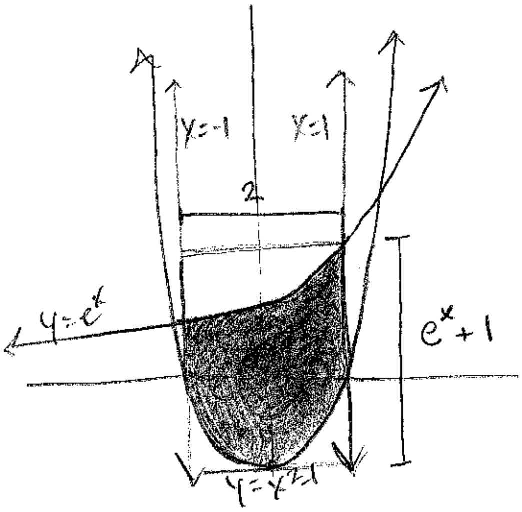
$$\int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx > \int_1^\infty \frac{x}{\sqrt{x^4-x}}\,dx > \int_1^\infty \frac{x}{\sqrt{x^4}}\,dx = \underbrace{\int_1^\infty \frac{1}{x}\,dx}_{\text{divergent by P-test } (P \le 1)}$$

Since $\int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx$ is greater than $\int_1^\infty \frac{1}{x}\,dx$, then $\int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx$ diverges also.

Solutions to 6.1 questions:

Erin Kendrick

5. $y=e^x$, $y=x^2-1$, $x=1$, $x=-1$



$$\int_{-1}^{1}\left[e^x-(x^2-1)\right]dx$$

$$\int_{-1}^{1}(e^x-x^2+1)\,dx$$

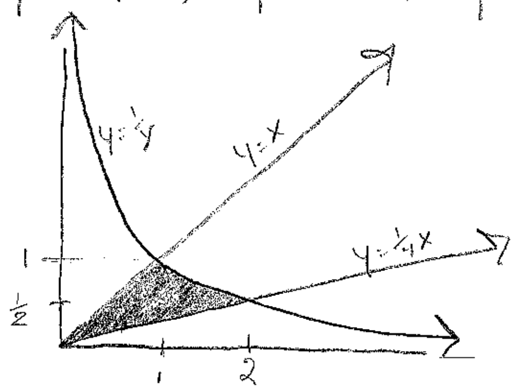
$$e^x-\tfrac{1}{3}x^3+x\Big]_{-1}^{1} = \left(e-\tfrac{1}{3}+1\right)-\left(\tfrac{1}{e}+\tfrac{1}{3}-1\right)$$

$$= e-\tfrac{2}{3}-\tfrac{1}{e}+2$$

$$= \boxed{e-\tfrac{1}{e}+\tfrac{4}{3}}$$

---

17. $y=1/x$, $y=x$, $y=\frac{1}{4}x$, $x>0$



$x=\frac{1}{y}$

$x=y$

$x=4y$

$$\int_0^{1/2}(4y-y)\,dy+\int_{1/2}^{1}\left(\tfrac{1}{y}-y\right)dy$$

$$=\int_0^{1/2}(3y)\,dy+\int_{1/2}^{1}\left(\tfrac{1}{y}-y\right)dy$$

$$\tfrac{3}{2}y^2\Big]_0^{1/2}+\ln|y|-\tfrac{1}{2}y^2\Big]_{1/2}^{1}$$

$$\tfrac{3}{2}\left(\tfrac{1}{4}\right)+\left(-\tfrac{1}{2}-\left(\ln\left(\tfrac{1}{2}\right)-\tfrac{1}{8}\right)\right)$$

$$=\tfrac{3}{8}-\tfrac{1}{2}+\ln(2)+\tfrac{1}{8}$$

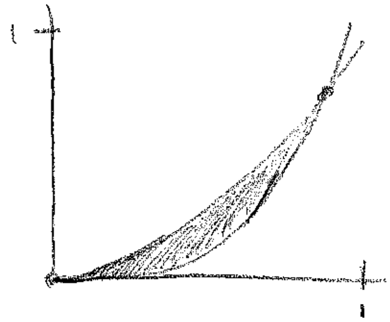
$$=\tfrac{1}{2}-\tfrac{1}{2}+\ln(2)$$

$$=\boxed{\ln(2)}$$

6.1 Solutions continued :

Erin Kendrick

19. $y = x\sin(x^2)$, $y = x^4$



intersections : $(0,0)$ & $(0.896, 0.644)$

$\boxed{x = 0, 0.9}$

$$\int_0^{0.896} \left[x\sin(x^2) - x^4\right] dx$$

$$\int_0^{0.896} x\sin(x^2)\,dx - \int_0^{0.896} x^4\,dx$$

$u = x^2$

$du = 2x\,dx$

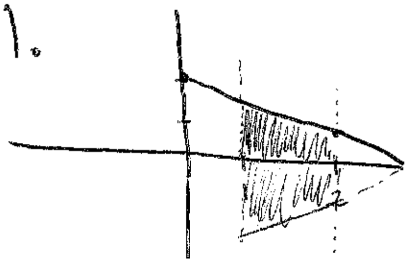
$$\frac{1}{2}\int_0^{(0.896)^2} \sin u\,du - \int_0^{0.896} x^4\,dx$$

$$-\frac{1}{2}\cos u\Big]_0^{.803} - \frac{1}{5}x^5\Big]_0^{0.896}$$

$= -0.3473 + \frac{1}{2} - 0.1155$

$= \boxed{0.04}$

1.



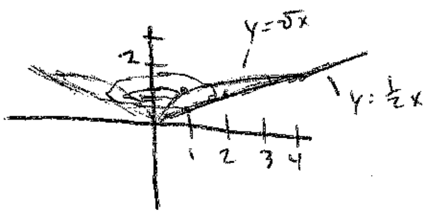
$y = 2 - \frac{1}{2}x$ $\quad y = 0$

$x = 1 \quad x = 2$

about x-axis

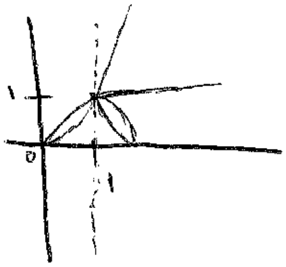
$$A = \pi r^2$$
$$= \pi(2 - \tfrac{1}{2}x)^2 \,\Delta x$$
$$= \pi\left(\tfrac{x^2}{4} - 2x + 4\right) \Delta x$$
$$V = \int_1^2 \pi\left(\tfrac{x^2}{4} - 2x + 4\right) \Delta x$$
$$= \pi\left[\frac{x^3}{4(3)} - \frac{2x^2}{2} + 4x\right]_1^2$$
$$= \pi\left[\left(\frac{8}{12} - 4 + 8\right) - \left(\frac{1}{12} - 1 + 4\right)\right]$$
$$= \pi\left[\frac{56}{12} - \frac{37}{12}\right]$$
$$= \boxed{\frac{19\pi}{12}}$$

7. $y^2 = x \qquad x = 2y \quad \Rightarrow \quad y = \sqrt{x} \qquad y = \frac{1}{2}x$ $\qquad$ about y-axis



$$A = \pi r^2$$
$$= \pi(2y)^2 - \pi(y^2)^2$$
$$= \pi(4y^2 - y^4)$$
$$V = \pi(4y^2 - y^4)\,\Delta y$$
$$= \int_0^2 \pi(4y^2 - y^4)\,dy$$
$$= \pi\left[\frac{4y^3}{3} - \frac{y^5}{5}\right]_0^2$$
$$= \pi\left(\frac{4(8)}{3} - \frac{32}{5}\right)$$
$$= \pi\left(\frac{160}{15} - \frac{96}{15}\right)$$
$$= \pi\,\frac{64}{15}$$

17. $y = x^3$, $y = \sqrt{x}$ about $x = 1$



$x = \sqrt[3]{y} = \underline{y^{1/3}}$

$x = \underline{y^2}$

$A = \pi r^2$

$= \pi(1-y^2)^2 - \pi(1-y^{1/3})^2$

$= \pi(y^4 - 2y^2 + 1) - \pi(y^{2/3} - 2y^{1/3} + 1)$

$= \pi(y^4 - 2y^2 - y^{2/3} + 2y^{1/3})$

$V = \pi \int_0^1 (y^4 - 2y^2 - y^{2/3} + 2y^{1/3})\, dy$

$= \pi \left[\frac{y^5}{5} - \frac{2y^3}{3} - \frac{3}{5} y^{5/3} + 2\left(\frac{3}{4}\right) y^{4/3} \right]_0^1$

$= \pi \left(\frac{1}{5} - \frac{2}{3} - \frac{3}{5} + \frac{3}{2} \right)$

$= \boxed{\pi \, \frac{13}{30}}$

6.4-6.5 Solutions   Peter Twomey

1) $L=\int_a^b \sqrt{\left(\frac{dx}{dy}\right)^2+1}\,dy \quad \frac{dx}{dy}=\frac{3}{2}y^{1/2}$

$L=\int_0^1 \sqrt{\frac{9}{4}y+1}\,dy$

$\left[\text{Table 21: } \int \sqrt{ax+b}\,dx=\left(\frac{2b}{3a}+\frac{2x}{3}\right)\sqrt{ax+b}\right]$

$L=\left(\frac{2(1)}{3\left(\frac{9}{4}\right)}+\frac{2y}{3}\right)\sqrt{\frac{9}{4}y+1}\,\Big]_0^1=\left(\frac{8}{27}+\frac{2y}{3}\right)\sqrt{\frac{9}{4}y+1}\,\Big]_0^1$

$=\left[\left(\frac{8}{27}+\frac{2}{3}\right)\sqrt{\frac{9}{4}+1}\right]-\frac{8}{27}\approx\boxed{1.44}$

2) $L=\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt \quad \frac{dx}{dt}=1-\sin t \quad \frac{dy}{dt}=1-\cos t$

$L=\int_0^{2\pi} \sqrt{(1-\sin t)^2+(1-\cos t)^2}\,dt=\int_0^{2\pi}\sqrt{1-2\sin t+\sin^2 t+1-2\cos t+\cos^2 t}\,dt$ Note: $\sin^2 t+\cos^2 t=1$

$=\boxed{\int_0^{2\pi}\sqrt{3-2\sin t-2\cos t}}$

3) $f(c)=f_{ave}=\frac{1}{b-a}\int_a^b f(x)\,dx$

$f_{ave}=\frac{1}{4-0}\int_0^4 4x-x^2\,dx=\frac{1}{4}\left[2x^2-\frac{x^3}{3}\right]_0^4=\frac{1}{4}\left[2(4)^2-\frac{4^3}{3}\right]=\frac{1}{4}\left[\frac{96}{3}-\frac{64}{3}\right]=\boxed{\frac{8}{3}}$

Cody Williams

Section 6.6 Springs & Cables

5. 10lb is required to hold Spring 4 in. from natural length How much work is done Stretching it 6 in. beyond natural length from it's natural length?

$4 \text{ in.} = 0.1016 \text{ m}$

$6 \text{ in.} = 0.1524 \text{ m}$

$f(x) = kx \qquad 10 = (0.1016)k$

$W = \int kx \qquad k = 98.425$

$$W = \int_0^{0.1524} (98.425)x \, dx = 49.212x^2 \Big|_0^{0.1524}$$

$$= 49.212\left((0.1524)^2 - (0)^2\right) = \boxed{1.1429 \text{ J}}$$

8. 12 ft-lb is required to stretch a Spring 1 ft beyond Natural length. How much work is needed to stretch it 9 in. beyond its natural length?

$9 \text{ in} = 0.75 \text{ ft}$

$f(x) = kx \qquad 12 = (1)k$

$W = \int kx \qquad k = 12$

$$W = \int_0^{0.75} 12x \, dx = 6x^2 \Big|_0^{0.75}$$

$$= 12\left((0.75)^2 - (0)^2\right) = \boxed{6.75 \text{ ft-lb}}$$

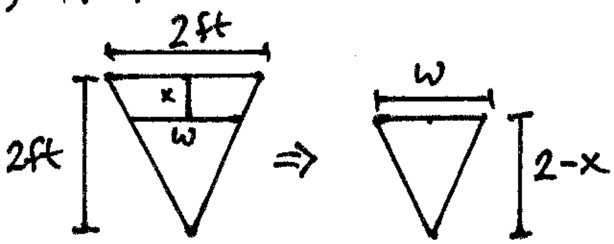
Christian [illegible]

6.6 Solutions

Steps

1. Find the Area of a cross-section
2. Solve for width (w)
   - w can be found by comparing its relation w/ known values
3. Find Volume (V) Δx or Δy
4. Find Displacement
   - distance from the surface of the fluid to the top of the container
5. Find Force → × by density
6. Work ... Integrate

1) $A = lw = 5w$



$\therefore w = 2-x$

$\therefore A = 5(2-x)$

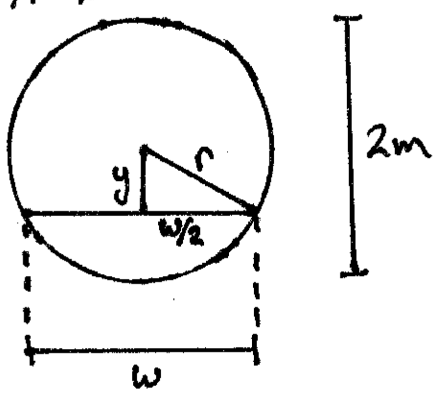
$V = lwh = 5(2-x)\Delta x$

Displacement $= x + \text{pipe} = x + 1.5\text{ ft}$

$F = (62.4)(5)(2-x)(x+1.5)\Delta x$

$W = \int_0^2 (62.4)(5)(2-x)(x+1.5)\,dx$

2) $A = lw = 7w$



$\therefore r = 1$

so... $1^2 = y^2 + \left(\frac{w}{2}\right)^2$

$\left(\frac{w}{2}\right)^2 = 1 - y^2$

$w = 2\sqrt{1-y^2}$

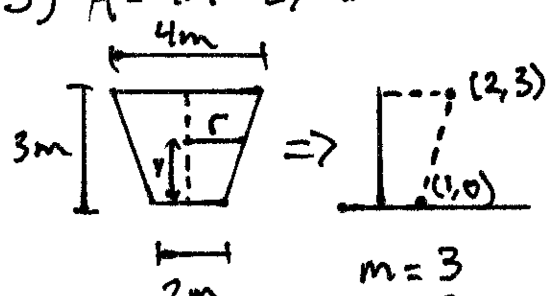
$\therefore V = 7(2\sqrt{1-y^2})\Delta y$

Displacement $= y + r = y + 1$

$F = (9.8 \times 1000)(7)(2\sqrt{1-y^2})(y+1)\Delta y$

$W = \int (9800)(7)(2\sqrt{1-y^2})(y+1)\,dy$

3) $A = \pi r^2 \Rightarrow \pi(y/3+1)^2$



$m = 3$

$y - 0 = 3(x-1)$

$y = 3x - 3$

$x = y/3 + 1$

$x = r$

$V = \pi r^2 h = \pi(y/3+1)^2\Delta y$

* b/c "y" is now from the bottom to water surface instead of original height to water surface Displacement $= h - y = 3 - y$

$F = (9.8 \times 1000)(\pi)(y/3+1)^2(3-y)\cancel{dy}\,\Delta y$

$W = \int_0^3 (9800)(\pi)(y/3+1)^2(3-y)\,dy$

6.7 2

Shane

Demand Price

(500, 50) (560, 45)

a) $P = -\frac{1}{12}x + \frac{275}{3}$

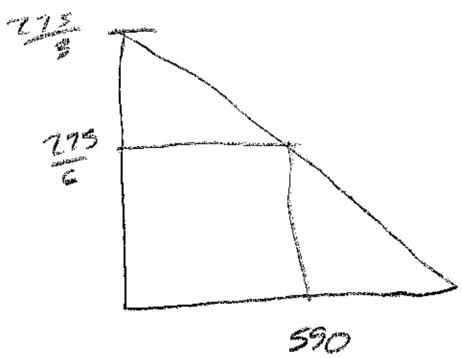
$R(x) = x \cdot P(x) = -\frac{1}{12}x^2 + \frac{275}{3}x$

$R'(x) = -\frac{1}{6}x + \frac{275}{3} = 0$

$x = 550$

$P(550) = \$45.83$

b) Consumer surplus



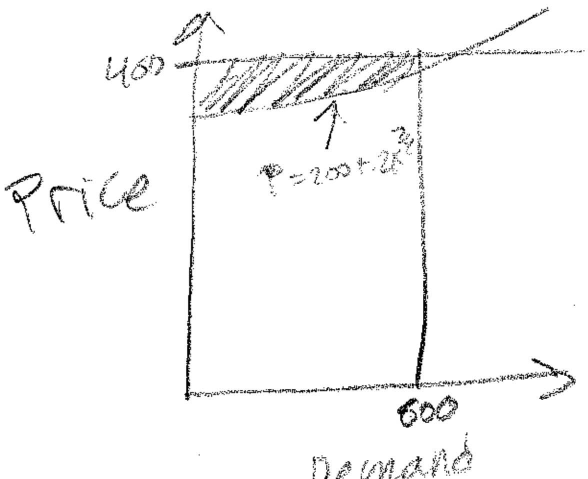
$\frac{1}{2} \cdot \frac{275}{6} \cdot 550 = \$12,604.17$

Answer

Shane Voss
6.7

7)

Supply $p = 200 + .2x^{3/2}$ \$400



$$400 = 200 + .2x^{3/2}$$

$$200 = .2x^{3/2}$$

$$X = 800$$

So $$\boxed{\int_0^{800} 400 - (200 + .2x^{3/2})\,dx}$$

$$\int_0^{800} 200 - .2x^{3/2}$$

$$200x - \frac{.2x^{5/2}}{\frac{3}{2}}\Bigg|_0^{800}$$

$$200(800) - \frac{(.2)(800)^{5/2}}{\frac{3}{2}} = \$12,000$$

6.7 (1)

Shane Voss

Answer

$P = 1200 - 0.2x - 0.001x^2$

$P(500) = 1075$

So

$$\int_0^{500} 1200 - .2x - .001x^2 - 1075$$

$$125x - \frac{.2}{2}x^2 - \frac{.001}{3}x^3 \Big|_0^{500}$$

$= \$33,333.33$

6.8 : Probability
Math 151-01
Kerry Nowacki

① Let $f(x) = .006x(10-x)$ for $0 \le x \le 10$
and $f(x) = 0$ for all other values of x.

a.) verify that $f(x)$ is a probability density function:
when $0 \le x \le 10$:
$.006x(10-x) \ge 0$ so, $f(x) \ge 0$ for all values x.
Check that equation 2 is satisfied:

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{10} .006x(10-x)\,dx \Rightarrow .006\int_0^{10} 10x - x^2\,dx =$$

$$.006\left[5x^2 - \frac{x^3}{3} + C\right]_0^{10} \Rightarrow .006\left[500 - \frac{1000}{3}\right] = 1$$

b.) Find $P(2 \le x \le 7)$:

$$P(2 \le x \le 7) = \int_2^7 .006x(10-x)\,dx = .006\int_2^7 (10x - x^2)\,dx$$

$$.006\left[5x^2 - \frac{x^3}{3}\right]_2^7 = 0.68$$

② let $f(x) = \frac{c}{(4+x^2)}$

For what values of c is f a probability density function:

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \frac{c}{4+x^2}\,dx \to c\int_{-\infty}^{\infty} \frac{1}{4+x^2}\,dx = \left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_{-\infty}^{\infty} = \frac{1}{c}$$

$$\tfrac{1}{2}\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \frac{1}{c}$$

$$\frac{\pi}{2} = \frac{2}{c} \Rightarrow \boxed{c = \frac{4}{\pi}}$$

6.8 Probability
Math 151-01
Kerry Nowacki

(3) The manager at Aromas determines that the average waittime is 2.5 minutes.

a) find the probability that a customer must wait more than 4 minutes:

$$f(t) = \begin{cases} 0 & \text{if } t<0 \\ \dfrac{e^{-t/\mu}}{\mu} & \text{if } t \geq 0 \end{cases} \Rightarrow f(t) = \begin{cases} 0 & \text{if } t<0 \\ \dfrac{e^{-t/2.5}}{2.5} & t \geq 0 \end{cases}$$

μ = mean waiting time

So: $\int_4^\infty f(t)dt = \int_4^\infty \frac{e^{-.4t}}{2.5} dt = \left[-e^{-.4t}\right]_4^\infty = .2$ = probability a customer would wait 4+ minutes.

b) Find the Probability a customer is served within the first 2 minutes.

$$\int_0^2 \frac{e^{-.4t}}{2.5} dt = \left[-e^{-.4t}\right]_0^2 = -e^{-.8} - (-e^0) = 1 - e^{-.8} = .55$$ probability they'll be served in the first 2 min.

c.) If the manager wants to give away a free burger to anyone who waits more than a certain amount of time, but doesn't want to give away to more than 2% of customers, How long would the customer have to wait?

$$\int_x^\infty .4e^{-.4t} dt = .02$$

$$= \left[-e^{-.4t}\right]_x^\infty = .02$$

$$\ln(e^{-.4t}) = (.02)\ln$$

$$-.4t = \ln|.02|$$

$$t = \frac{\ln|.02|}{-.4} = 9.780 \text{ minutes.}$$

Evie
Blackburn

Section 7.1

solutions

1. $y = \frac{2}{3} e^{x} + e^{-2x}$ and $y' + 2y = 2e^{x}$

$y' = \frac{2}{3} e^{x} - 2x e^{-2x}$

$\left(\frac{2}{3} e^{x} - 2x e^{-2x}\right) + 2\left(\frac{2}{3} e^{x} + e^{-2x}\right) = 2e^{x}$

$= \frac{2}{3} e^{x} \cancel{- 2x e^{-2x}} + \frac{4}{3} e^{x} \cancel{+ 2e^{-2x}} = 2e^{x}$

$= \frac{6}{3} e^{x} = 2e^{x}$

$= 2e^{x} = 2e^{x}$ ✓

5. $y'' + y = \sin(x)$

a. $y = \sin(x)$
$y' = \cos(x)$
$y'' = -\sin(x)$

$-\sin(x) + \sin(x) = \sin(x)$ X

b. $y = \cos(x)$
$y' = -\sin(x)$
$y'' = -\cos(x)$

$-\cos(x) + \cos(x) = \sin(x)$ X

c. $y = \frac{1}{2} x \sin(x)$
$y' = \frac{1}{2} \sin(x) + \cos(x) \frac{1}{2}(x)$
$y'' = \frac{1}{2}\cos(x) + \frac{1}{2}\cos(x) - \sin(x)\frac{1}{2}x$

$\frac{1}{2}\cos(x) + \frac{1}{2}\cos(x) \cancel{- \sin(x)\frac{1}{2}x} \cancel{+ \frac{1}{2}x\sin(x)} = \sin(x)$

$\cos(x) = \sin(x)$ X

d. $y = \frac{1}{2} x \cos(x)$
$y' = \frac{1}{2} x \sin(x) - \frac{1}{2}\cos(x)$
$y'' = \frac{1}{2}\sin(x) + \frac{1}{2}x\cos(x) + \frac{1}{2}\sin(x)$

$\frac{1}{2}\sin(x) \cancel{+ \frac{1}{2}x\cos(x)} + \frac{1}{2}\sin(x) \cancel{- \frac{1}{2}x\cos(x)} = \sin(x)$

$\sin(x) = \sin(x)$ ✓

d. $y = \frac{1}{2} x \cos(x)$ is a solution to $y'' + y = \sin(x)$

Section 7.1
Solutions

Evie Blackburn

9. $\frac{dP}{dt} = 1.2P\left(1 - \frac{P}{4200}\right)$

a. $0 < P < 4200$

because the derivative $\left[\frac{dP}{dt}\right]$ is positive within these values.

b. $P > 4200$

because the derivative $\left[\frac{dP}{dt}\right]$ is negative within these values.

c. $P = 0$, $P = 4200$

because the population equals 0 at these points.

Elizabeth Petty
8.1

1) $a_n = \frac{3^{n+2}}{5^n}$ $r = \left(\frac{3}{5}\right)$

$-1 < r < 1$, so the series converges

$$\lim_{n \to \infty} \frac{3^{n+2}}{5^n} = 0$$

2) $a_n = \frac{\sin 2n}{1+\sqrt{n}}$

Comparison test: $\frac{\sin 2n}{1+\sqrt{n}} < \frac{3}{1+\sqrt{n}} < \frac{3}{n^{\frac{1}{2}}}$

by the P-test $\frac{3}{n^{\frac{1}{2}}}$ converges, so $\frac{\sin 2n}{1+\sqrt{n}}$ also converges

$$\lim_{n \to \infty} \frac{\sin 2n}{1+\sqrt{n}} = 0$$

3) (a) $a_n = 1060, 1123.6, 1191, 1262.5, 1338.2$

(b) Divergent because $r > 1$

$\underline{8.2: \text{Series}}$

Katelin Maatz

1. $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}}$

$= \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{\pi^n}{3^n} = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)\left(\frac{\pi}{3}\right)^n$

$a = \frac{1}{3}$ $r = \frac{\pi}{3} > 1$ $\Rightarrow$ the series diverges because $r > 1$

2. $\sum_{n=1}^{\infty} \frac{1 + 2^n}{3^n}$

$= \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

$a = \frac{1}{3}$ $r = \frac{1}{3} < 1$ $a = \frac{2}{3}$ $r = \frac{2}{3} < 1$ $\Leftarrow$ for both sums, $r < 1$ so the series converges

$\frac{a}{1-r}$ $\frac{\frac{1}{3}}{1 - \frac{1}{3}}$ $\frac{\frac{2}{3}}{1 - \frac{2}{3}} = \frac{\frac{2}{3}}{\frac{1}{3}} = \frac{2}{3} \cdot \frac{3}{1} = \boxed{2}$

$= \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{3} \cdot \frac{3}{2} = \boxed{\frac{1}{2}}$

$\frac{1}{2} + 2 = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$

Series converges to $\frac{5}{2}$

3. $1.53\overline{42}$

$= 1.53 + \underbrace{\frac{42}{10^3} + \frac{42}{10^5} + \frac{42}{10^7} + \ldots}_{}$ $= 1.53 +$

geometric series: $a = \frac{42}{10^3}$ $r = \frac{1}{10^2} < 1 \Rightarrow$ converges

$\frac{a}{1-r} = \frac{\frac{42}{1000}}{1 - \frac{1}{100}} = \frac{\frac{42}{1000}}{\frac{99}{100}} = \frac{42}{1000} \cdot \frac{100}{99} = \frac{42}{990}$

$= \frac{153}{100} + \frac{42}{990} = \frac{5189}{3300}$

8.3 The Integral and Comparison Tests; Estimating Sums

Maddy Horn Solutions:

1) $\sum_{n=1}^{\infty} n^b$ is the P-series. It converges when $b<-1$
when written $\frac{1}{n^p}$ converges when $p>1$

$\sum_{n=1}^{\infty} b^n$ geometric series converges when $-1<b<1$
can think of it as converges when $|r|<1$
when $r=b$

2) $\sum_{n=1}^{\infty} \frac{(\cos n)^2}{n^2+1}$ $\quad 0 \le \cos^2 n \le 1$ $\quad n^2+1 > n^2$

$0 \le \frac{\cos^2 n}{n^2+1} \le \frac{1}{n^2+1}$ $\quad \frac{1}{n^2+1} < \frac{1}{n^2}$

by p-series test $\sum \frac{1}{n^2}$ is convergent. By comparison test to $\sum \frac{1}{n^2}$ the series $\sum \frac{1}{n^2+1}$ and the original series $\sum \frac{\cos^2 n}{n^2+1}$ are also convergent.

3) $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$ $\quad 1 \le 2+(-1)^n \le 3$ $\quad \frac{1}{n^{3/2}} \le \frac{2+(-1)^n}{n\sqrt{n}} \le \frac{3}{n^{3/2}}$

$$\frac{3}{n^{3/2}} \rightsquigarrow \frac{3}{x^{3/2}}\; 3\int_1^{\infty} x^{-3/2} = 3\left[\lim_{B\to\infty} \frac{-2}{\sqrt{x}}\Big|_1^B\right] = 3\cdot -2 = -6$$

convergent

by integral test $\sum \frac{3}{n^{3/2}}$ is convergent thus by comparison test $\sum \frac{2+(-1)^n}{n\sqrt{n}}$ is also convergent.

8.4 Kelly Wilson

1.) Approximate the sum to 4 decimals

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{8^n} = -\frac{1}{8} + \frac{2}{64} - \frac{3}{512} + \frac{4}{4096} - \frac{5}{32768} + \frac{6}{262{,}144} \ldots$$

$$S - S_6 \leq |R_6|$$

$$R_6 = \frac{6}{262{,}144} \approx .000023$$

$$-\frac{1}{8} + \frac{2}{64} - \frac{3}{512} + \frac{4}{4096} - \frac{5}{32768} \approx -.098785$$

$$\boxed{-.098785}$$

8.4 Kelly Wilson

2.) Determine whether the series is absolutely convergent

$$\sum_{n=0}^{\infty} \frac{(-10)^n}{n!} = -10 + \frac{100}{2!} - \frac{-1000}{3!} \dots$$

$$\sum_{n=0}^{\infty} \left| \frac{(-10)^n}{n!} \right| = 10 + \frac{100}{2!} + \frac{1000}{3!} \dots$$

$$\downarrow$$

$$\sum_{n=0}^{\infty} \frac{10^n}{n!} \xrightarrow{\text{Ratio test}} \lim_{n \to \infty} \frac{(10^{n+1})(n!)}{(n+1)!\,(10^n)} =$$

$$\lim_{n \to \infty} \frac{10}{n+1} = 10 \lim_{n \to \infty} \frac{1}{n+1} = 0 \qquad 0<1$$

$0<1$, therefore by the ratio test $\sum_{n=0}^{\infty} \frac{(-10)^n}{n!}$ is absolutely convergent

8.4 Kelly W

3.) Determine if the series is convergent, absolutely convergent, or divergent

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$$

Alternating series test

$\frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}} \Longrightarrow$ Therefore the series is decreasing

$\lim_{n \to \infty} \frac{1}{\sqrt{n}} = \lim_{n \to \infty} \frac{1}{n^{1/2}} \neq 0$

$\frac{1}{n^{1/2}}$ diverges by the p-series test, because $1/2 \leq 1$.

Therefore, by alternating series test, the series diverges.

8.5 Power Series Solution 1

Brooke Zalud

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$$

$$a_n = \frac{(-1)^n x^n}{n+1}$$

ratio test $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(-1)^{n+1} x^{n+1}}{n+2} \cdot \frac{n+1}{(-1)^n x^n}\right| = \frac{|x|(n+1)}{n+2}$

$$|x| \lim_{n\to\infty} \frac{n+1}{n+2} = 1 \cdot |x|$$

$$-1 < x < 1$$

ROC $= 1$

IOC $= (-1, 1)$

8.9 Power Series Solution 2

Brooke Zalud

$$\sum_{n=1}^{\infty} \frac{(2n)!}{2^n} \cdot x^n$$

$$a_n = \frac{(2n)!}{2^n} \cdot x^n$$

ratio test $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(2n+1)!}{2^{n+1}} \cdot x^{n+1} \cdot \frac{2^n}{(2n)! \cdot x^n}\right|$

$$= \frac{(2n+1)(2n+2)\ x}{2}$$

$$x \cdot \lim_{n \to \infty} \frac{(2n+1)(2n+2)}{2}$$

$$x \cdot \quad \infty$$

$$ROC = 0$$

$$IOC = 0$$

8.9 Power Series Solution 3

Brooke Zalud

$$\sum_{n=1}^{\infty} n!\,(2x-1)^n$$

$a_n = n!\,(2x-1)^n$

ratio test $\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)!\,(2x-1)^{n+1}}{n!\,(2x-1)^n}$

$= (n+1)(2x-1)$

$(2x-1) \cdot \lim_{n\to\infty} n+1$

$2x-1 < 0$

$x = \frac{1}{2}$

ROC = 0

IOC = $x = 1/2$

Math 151 Review Problems ~ Section 8.6 (Ryann Garcia) $\boxed{\text{Answers}}$

#1) Find a power series representation for the function and determine the interval of convergence.

$$f(x) = \frac{x}{9+x^2}$$

$$f(x) = x\left(\frac{1}{9+x^2}\right)$$

$$f(x) = x\left(\frac{1}{9-(-x^2)}\right)$$

$$f(x) = x\left(\frac{1}{9\left(1-\left(-\frac{x^2}{9}\right)\right)}\right)$$

$$f(x) = \frac{x}{9}\left(\frac{1}{1-\left(-\frac{x^2}{9}\right)}\right)$$

Because we know that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, then

$$= \frac{x}{9}\sum_{n=0}^{\infty}\left(\frac{-x^2}{9}\right)^n$$

$$= \frac{x}{9}\sum_{n=0}^{\infty}\frac{(-x^2)^n}{9^n}$$

$$= \frac{x}{9}\sum_{n=0}^{\infty}(-1)^n\frac{(x^2)^n}{9^n}$$

$$= \frac{x}{9}\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{9^n}$$

$$= \boxed{\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{9^{n+1}}}$$

Interval of Convergence

*Ratio Test

$$\lim_{x\to\infty}\left|\frac{(-1)^{n+1}x^{2n+3}}{9^{n+2}}\cdot\frac{9^{n+1}}{(-1)^n x^{2n+1}}\right|$$

$$\lim_{x\to\infty}\left|\frac{(-1)x^2}{9}\right|$$

$$\lim_{x\to\infty}\left|\frac{x^2}{9}\right| = \frac{x^2}{9}$$

$$-1 < \frac{x^2}{9} < 1$$

$$x^2 < 9$$

$$x < \pm 3 \rightarrow -3 < x < 3$$

$\boxed{\text{Interval of convergence } (-3,3)}$

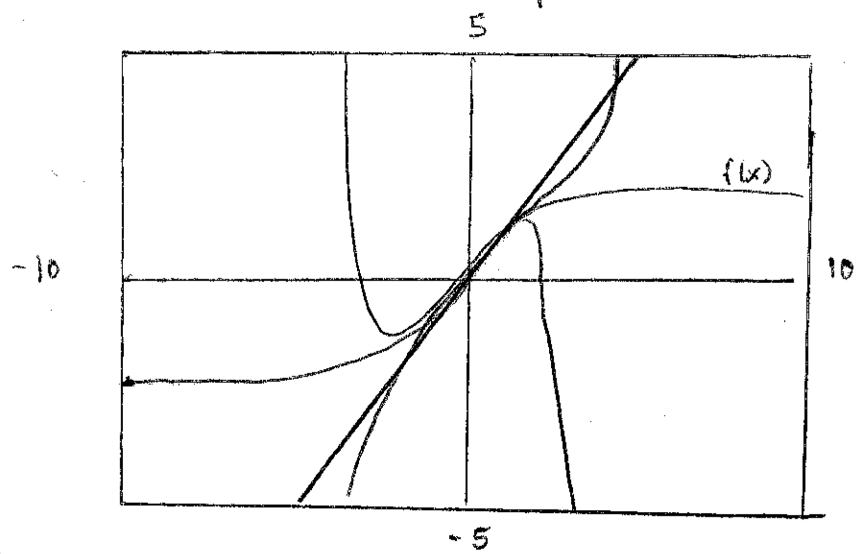
Math 151 Review Problems ~ Section 8.6 (Ryann Garcia) **Answers**

#2 Find a power series representation for f, and graph f and several partial sums $S_n(x)$ on the same screen. What happens as n increases?

$f(x) = \tan^{-1}(2x)$

Because we know that $\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{2n+1}$, then

$$\boxed{\begin{aligned} f(x) &= \tan^{-1}(2x) \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{2n+1} = 2x - \frac{(2x)^3}{3} + \frac{(2x)^5}{5} - \frac{(2x)^7}{7} + \dots \end{aligned}}$$



As n increases, the graphs of the partial sums begin to fit f(x) much better.

#3 Use a power series to approximate the definite integral to six decimal places.

$\int_0^{0.2} \frac{1}{1-x^3}\,dx \Rightarrow f(x) = \frac{1}{1-x^3}$; because we know that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, then

$$f(x) = \frac{1}{1-x^3} = \sum_{n=0}^{\infty} (x^3)^n \Rightarrow \sum_{n=0}^{\infty} x^{3n}$$

$$\int_0^{0.2} x^{3n}\,dx = \frac{x^{3n+1}}{3n+1} = \left[x + \frac{x^4}{4} + \frac{x^7}{7} + \frac{x^{10}}{10} + \dots \right]_0^{0.2} = \underset{S_0}{0.2} + \underset{S_1}{\frac{(0.2)^4}{4}} + \underset{S_2}{\frac{(0.2)^7}{7}} + \underset{S_3}{\frac{(0.2)^{10}}{10}} + \dots$$

$\frac{x^{10}}{10} < 0.000001$

$\boxed{S_2 = 0.200402}$

JACK ORR

SECTION 8.7

TAYLOR AND MACLAURIN SERIES

1) $f(x) = \cos x \,,\ a = \pi$

TAYLOR SERIES IS $\left\{ \sum_{n=0}^{\infty} \frac{f^{n}(a)(x-a)^n}{n!} = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \ldots \right.$

FIND $\frac{d}{dx}$'S OF $f(x)$

$f(x) = \cos x \quad f(\pi) = -1$

$f'(x) = -\sin x \quad f'(\pi) = 0$

$f''(x) = -\cos x \quad f''(\pi) = 1$

$f'''(x) = \sin x \quad f'''(\pi) = 0$

FILL IN TAYLOR SERIES

$$-1 + \frac{0(x-\pi)}{1!} + \frac{(x-\pi)^2}{2!} + \frac{0(x-\pi)^3}{3!} - \frac{(x-\pi)^4}{4!} + \ldots$$

SIMPLIFY AND INTERPRET

$$-1 + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-\pi)^{2n}}{(2n)!}$$

COSX ROC IS ∞ SO ROC IS ∞

JACK ORR

SECTION 8.7

TAYLOR AND MACLAURIN SERIES

EVALUATE $\lim_{x \to 0} \dfrac{\sin x - x + \frac{1}{6}x^3}{x^5}$

WE KNOW

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

SUB THIS INTO THE LIMIT

$$\lim_{x \to 0} \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots - x + \frac{1}{6}x^3}{x^5}$$

SIMPLIFY

$$\lim_{x \to 0} \frac{-\frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + \frac{1}{6}x^3}{x^5} \Rightarrow \lim_{x \to 0} \frac{+\frac{x^5}{5!} - \frac{x^7}{7!} + \dots}{x^5}$$

DIVIDE BY HIGHEST POWER OF X IN DENOMINATOR

$$\lim_{x \to 0} \frac{1}{5!} - \frac{x^2}{7!} + \dots \Rightarrow \frac{1}{5!} = \frac{1}{120}$$

JACK ORR

SECTION 8.7

TAYLOR AND MACLAURIN SERIES

APPROXIMATE $\int_0^{0.3} \tan^{-1}(x^3)\,dx$ TO 5 DECIMAL PLACES

WE KNOW

$$\tan^{-1}x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$$

$$\text{SO } \tan^{-1}(x^3) = x^3 - \frac{x^9}{3} + \frac{x^{15}}{5} - \frac{x^{21}}{7} + \frac{x^{27}}{9} - \cdots$$

INTIGRATE EACH TERM

$$\int x^3 - \frac{x^9}{3} + \cdots = \frac{x^4}{4} - \frac{x^{10}}{27} + \frac{x^{16}}{80} - \frac{x^{22}}{154} + \frac{x^{28}}{252} - \cdots \Bigg]_0^{0.3}$$

PLUG IN LIMITS OF INTIGRATION

$$\frac{(0.3)^4}{4} - \frac{(0.3)^{10}}{27} + \frac{(0.3)^{16}}{80} - \frac{(0.3)^{22}}{154} + \frac{(0.3)^{28}}{252} - \cdots$$

$$= 0.00202$$

Matteo Salam 8.7

(answers)

1.) $f(x) = x^2(\ln(1+x^3))$

$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ (table)

$x = x^3$

$$x^2 \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x^3)^n}{n} = \boxed{\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{3n+2}}{n}}$$

2.) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ (table)

$x = -0.2$

$$e^{-0.2} = \sum_{n=0}^{\infty} \frac{(-0.2)^n}{n!} = \underbrace{1 - 0.2 + 0.02 + 0.00007}_{S_3} - 0.000003$$

$S_3 = \boxed{0.81877}$

3.) $1 - \ln(2) + \frac{\ln(2)^2}{2!} - \frac{\ln(2)^3}{3!} + \dots$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(\ln 2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-\ln 2)^n}{n!}$$ where $x = -\ln(2)$

$$= e^x \longrightarrow e^{-\ln 2} = \frac{1}{e^{\ln 2}}$$

$$= \boxed{\frac{1}{2}}$$