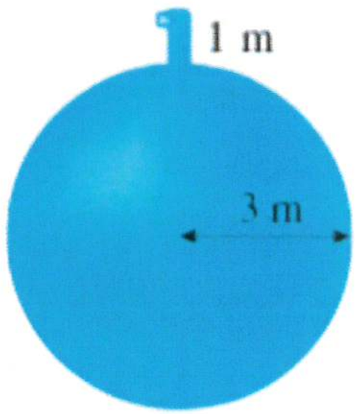
Mendel Chan

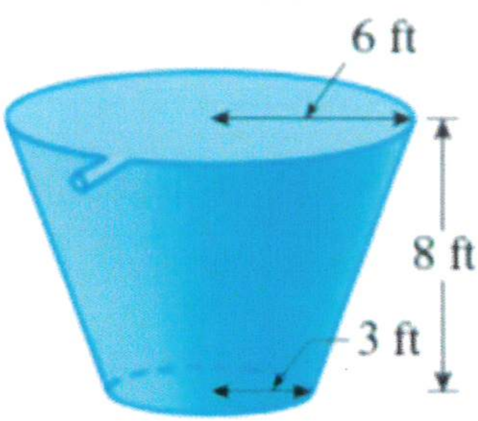
A tank is full of water. Find the work required to pump the water out of the spout. Use the fact that water weighs 62.5 lb/ft^3.

1)



sphere

2)



frustum of a cone

Mendel Chan

1)

$A_{xs} = \pi(\sqrt{9-y^2})^2$ $\quad g = 9.8$

$1000.$

$= \pi(9-y^2)$

$V_{slice} = \pi(9-y^2)\Delta y$

$F_{slice} = \pi(9-y^2)\Delta y(62.5\,lb/ft^3)$

$W = \pi(9-y^2)\Delta y(62.5\,lb/ft^3)(4+y)$

$(9-y^2)(4+y)$

$36+9y-4y^2-y^3$

$= 980\pi \int_{-3}^{3} (9-y^2)(4+y)\,dy$

$= 980\pi \int_{-3}^{3} 36+9y-4y^2-y^3\,dy$

$= 980\pi\left(36y+\frac{9y^2}{2}-\frac{4y^3}{3}-\frac{y^4}{4}\right)$

$= 35280\pi x+4410\pi x^2-\frac{3920\pi x^3}{3}-245\pi x^4 \Big|_{-3}^{3}$

$= 141120\pi$

≈ 443342 m

2)

$\text{weight} = 62.5\,lb/ft^3$

$\begin{matrix} x & y \end{matrix}$
$(3,0)$
$(6,8)$

$A_x = \pi(r)^2$ $\quad r = \frac{3}{8}y+3$

$\pi(x)^2$ $\quad$ If $y=0$ $\quad y=8$

$\pi\left(\frac{3}{8}y+3\right)^2$ $\quad \frac{3}{8}(0)+3$ $\quad \frac{3}{8}(8)+3$

$x=3$ $\quad x=6$

$\pi\left(\frac{3y}{8}+3\right)^2$

$A_{xs} = \pi\left(\frac{9y^2}{64}+\frac{9y}{4}+9\right)$

$V_{xs} = \pi\left(\frac{9y^2}{64}+\frac{9y}{4}+9\right)\Delta y$

$F_{slice} = \pi\left(\frac{9y^2}{64}+\frac{9y}{4}+9\right)(62.5\,lb/ft^3)\Delta y$

$W_{slice} = \pi(62.5\,lb/ft)(8-y)\left(\frac{9y^2}{64}+\frac{9y}{4}+9\right)\Delta y$

$62.5\pi\int_0^8 (8-y)\left(\frac{9y^2}{64}+\frac{9y}{4}+9\right)dy$

$62.5\pi\int_0^8 \frac{9y^2}{8}+18y+72-\frac{9y^3}{64}-\frac{9y^2}{4}-9y\,dy$

$= 62.5\pi\int_0^8 -\frac{9y^2}{8}+9y+72-\frac{9y^3}{64}\,dy$

$= \left(-\frac{9y^3}{24}+\frac{9y^2}{2}+72y-\frac{9y^4}{256}\right)62.5\pi$

$= \frac{-12000\pi x^3+140000\pi x^2+2304000\pi x-1125\pi x^4}{512}\Big|_0^8$

$= 500\pi+528$

≈ 2100 ft/lb

Jacques Benzly Te Topic - 6.6 (springs/cables)

9.) A spring has a natural length of 20 cm. Compare work W_1 done in stretching the spring from 20 cm to 30 cm with the work done W_2 done in stretching it from 30 cm to 40 cm. How are W_1 and W_2 related.

13.) A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mine shaft 500 ft deep. Find the work done.

Jacques Benzly Te   Topic - 6.6 (Springs/cable)

9.) Solution: Natural length 20 cm (integrate difference in meters)

To find W_1: 20cm → 30cm   To find W_2: 30cm → 40cm

$F = KX$ $F = KX$

$W_1 = \int_0^{0.10} KX \, dX$ $W_2 = \int_{0.10}^{0.20} KX \, dX$

$W_1 = \frac{KX^2}{2} \Big|_0^{0.10}$ $W_2 = \frac{KX^2}{2} \Big|_{0.10}^{0.20}$

$\frac{K \cdot 0.1^2}{2} - \frac{K \cdot 0}{2}$ $W_2 = \frac{0.20^2}{2} \cdot K - \frac{0.10^2}{2} K$

$\boxed{W_1 = \frac{1}{200} K \text{ Joules}}$ $W_2 = \frac{1}{50} K - \frac{1}{200} K$

$$\boxed{W_2 = \frac{3}{200} K}$$

To find relation between W_1 and W_2 isolate and equate K

$200 W_1 = \frac{200}{3} W_2$ $\boxed{3 W_1 = W_2}$

$600 W_1 = 200 W_2$

Jacques Benzly Te Topic 6.6 (Springs/cables)

13.) Solution:

given lbs so no need to consider $F = ma$

$W = F\Delta X$

Cable weight $= \frac{2\,lb}{ft} \cdot 500\,ft = 1000\,lbs$

$W = \int F\,dX$

Cable weight + load weight $= 1000 + 800 = 1800\,lbs$

function for weight as coal is lifted

$W = \int 1800 - 2X\,dX$

$F(X) = (1800 - 2X)$

initial weight — weight reduced per ft

$W = \int_{0\,ft}^{500\,ft} 1800 - 2X\,dX$

$W = 1800X - \frac{2X^2}{2} \Big|_0^{500\,ft}$

$[1800[500] - 500^2] - [1800 \cdot 0 - \frac{20^2}{2}]$

$[650{,}000] \quad -[0]$

$\boxed{W = 650{,}000\ ft\text{-}lb}$

Maxwell Kelley MATH-151-01 CALC II | 6.4 non-Parametric |

① Find the arc length of $y = 2x - 5$, $-1 \leq x \leq 3$

② Find the arc length of $y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x)$, $1 \leq x \leq 2$

Maxwell Kelley MATH-151-01 Calc II 6.4 non-Parametric

① Find the arc length of $y = 2x - 5$, $-1 \leq x \leq 3$

$y' = 2$

formula to use:

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$L = \int_{-1}^{3} \sqrt{1 + (2)^2}\, dx$$

$$= \int_{-1}^{3} \sqrt{5}\, dx$$

$$= \sqrt{5} \cdot x + C \Big]_{-1}^{3}$$

$$= \sqrt{5}(3) - \sqrt{5}(-1)$$

$$= \sqrt{5}(3) + \sqrt{5}$$

$$\boxed{\begin{array}{l} L = 4\sqrt{5} \\ L \approx 8.944 \end{array}}$$

② Find arc length of $y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x)$, $1 \leq x \leq 2$

$$y' = \frac{x}{2} - \frac{1}{2x} = \frac{x^2 - 1}{2x}$$

formula to use:

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{x^2 - 1}{2x}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + \frac{x^4 - 2x^2 + 1}{4x^2}}$$

$$= \int_1^2 \sqrt{\frac{x^4 + 2x^2 + 1}{4x^2}}\, dx$$

$$= \int_1^2 \sqrt{\frac{x^2 + 2 + \frac{1}{x^2}}{4}}\, dx$$

$$= \int_1^2 \sqrt{\frac{1}{4}\left(x^2 + 2 + \frac{1}{x^2}\right)}\, dx$$

$$= \frac{1}{2}\int_1^2 \sqrt{\left(x + \frac{1}{x}\right)^2}\, dx$$

$$= \frac{1}{2}\int_1^2 \left(x + \frac{1}{x}\right) dx$$

$$= \frac{1}{2}\left(\frac{x^2}{2} + \ln(x)\right)\Big]_1^2$$

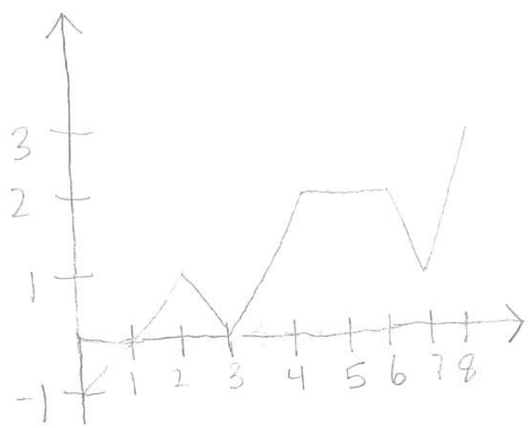
$$= \frac{1}{2}\left(\frac{2^2}{2} + \ln(2)\right) - \frac{1}{2}\left(\frac{1^2}{2} + \ln(1)\right)$$

$$\boxed{\begin{array}{l} L = \dfrac{3 + \ln(4)}{4} \\ L \approx 1.097 \end{array}}$$

Delaney Nolan

6.5 Problems

① Find the average value of the function $f(x) = 1 + x^2$ on the interval $[1, 2]$.

② 

Find the average value of f on the interval $[0, 8]$.

6.5

Delaney Nolan Solutions

(1) $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-(-1)}\int_{-1}^{2}(1+x^2)\,dx$

$= \frac{1}{3}\left[x+\frac{x^3}{3}\right]_{-1}^{2} = \frac{1}{3}\left[\left(2+\frac{2^3}{3}\right)-\left(-1+\frac{(-1)^3}{3}\right)\right]$

$= 2$

(2) Answer $= \frac{9}{8}$

David Espinoza

6.2 ANSWERS

(1)

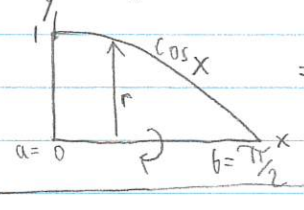
a) $\pi \int_0^{\pi/2} \cos^2 x \, dx$

$V_{disk} = \int_a^b A(x)\,dx = \int_a^b [\pi r^2]\,dx = \pi \int_a^b r^2\,dx \Rightarrow r^2 = \cos^2 x$

$\Rightarrow r = \cos x$

$a = 0$

$b = \frac{\pi}{2} \Rightarrow$



$\Rightarrow 0 \le y \le \cos x,\ 0 \le x \le \frac{\pi}{2}$; ABOUT THE X-AXIS

---

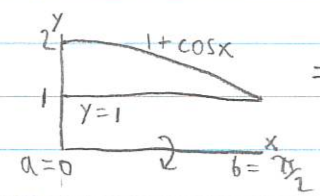
b) $\pi \int_0^{\pi/2} [(1+\cos x)^2 - 1]\,dx$

$V_{WASHER} = \int_a^b A(x)\,dx = \int_a^b [\pi(r_{out}^2 - r_{in}^2)]\,dx = \pi \int_a^b [r_{out}^2 - r_{in}^2]\,dx$

$\Rightarrow r_{out}^2 - r_{in}^2 = (1+\cos x)^2 - 1 \Rightarrow r_{out} = 1 + \cos x$; $r_{in} = \sqrt{1} = 1$

$a = 0$

$b = \frac{\pi}{2} \Rightarrow$



$\Rightarrow 1 \le y \le 1 + \cos x,\ 0 \le x \le \frac{\pi}{2}$; ABOUT THE X-AXIS

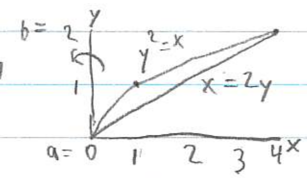
---

(2) $y^2 = x,\ x = 2y$; ABOUT THE Y-AXIS

FIND BOUNDS $\Rightarrow y^2 = 2y \Rightarrow y^2 - 2y = 0 \Rightarrow y(y-2) = 0 \Rightarrow \boxed{y = 0,\ y = 2}$

$\Rightarrow x = y^2 \Rightarrow \boxed{x = 0,\ x = 4}$

SKETCH REGION

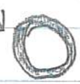


$\Rightarrow r_{out} = 2y \Rightarrow r_{out}^2 = 4y^2$

$r_{in} = y^2 \Rightarrow r_{in}^2 = y^4$

SKETCH SOLID:

SKETCH WASHER ↘



$\Rightarrow V = \int_a^b A(y)\,dy = \int_a^b \pi(r_{out}^2 - r_{in}^2)\,dy = \pi \int_0^2 (4y^2 - y^4)\,dy$

$= \pi \left[\frac{4y^3}{3} - \frac{y^5}{5}\right]_0^2 = \pi\left[\left(\frac{4(2^3)}{3} - \frac{2^5}{5}\right) - \left(\cancel{\frac{4(0^3)}{3}} - \cancel{\frac{0^5}{5}}\right)\right]$

$= \pi\left[\frac{32}{3} - \frac{32}{5}\right] = \frac{32(5-3)\pi}{3(5)} = \boxed{\frac{64\pi}{15}}$

David
Espinoza

6.2 VOLUMES

DISC AND WASHER METOD

① THE DEFINITE INTEGRAL REPRESENTS THE VOLUME OF A SOLID. DESCRIBE THE SOLID

a) $\pi \int_0^{\pi/2} \cos^2 x \, dx$

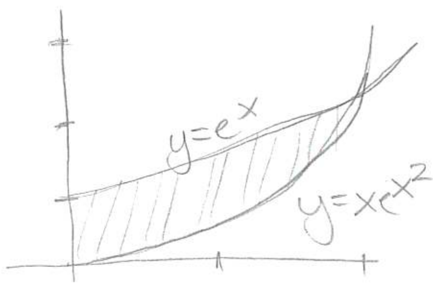
b) $\pi \int_0^{\pi/2} [(1+\cos x)^2 - 1] \, dx$

② FIND THE VOLUME OBTAINED BY ROTATING THE REGION BOUNDED BY THE GRAPHS OF THE GIVEN EXPRESSIONS ABOUT THE SPECIFIED LINE. SKETCH THE REGION, THE SOLID, AND A TYPICAL DISK OR WASHER:

$y^2 = x$, $x = 2y$; ABOUT THE Y-AXIS

Dominique Gillette 6.1

2. Find the area of the shaded region



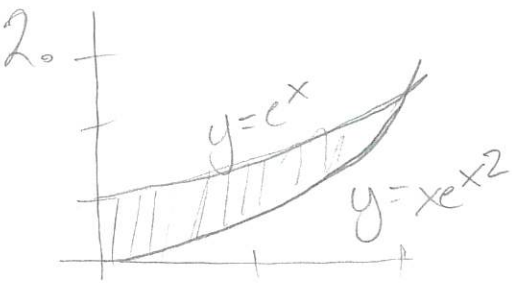
27. Sketch the region enclosed by the graphs of the given functions and find the area of the region.

$y=\frac{1}{4}x$, $y=\frac{1}{x}$, $y=x$, $x>0$

Dominique 6.1

Solutions

2.



since e^x is on top of of the shaded area that is what will be the positive part of the solution

$\int_0^1 e^x - xe^{x^2} dx$ u subsitution $\Rightarrow u = x^2$ $du = 2x dx$

$\frac{1}{2} du = x dx$

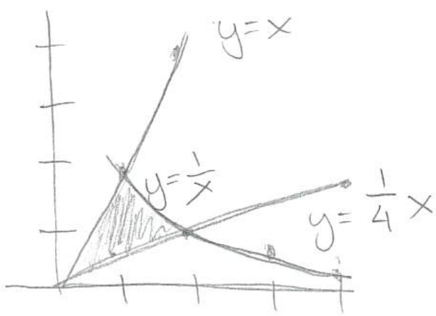
$\int e^x - \frac{1}{2}\int e^u du = \frac{1}{2}e^u$ plug x back in $= \frac{1}{2}e^{x^2}$

$\Rightarrow e^x - \frac{1}{2}e^{x^2}\Big]_0^1 = [e^1 - \frac{1}{2}e^1] - [e^0 - \frac{1}{2}e^0] = e - \frac{1}{2}e - 1 + .5$

$= \frac{e-1}{2}$

Dominique 6.1

26.



$y=y \quad \frac{1}{x}=x \quad x^2=1 \quad x=1$ since $x>0$

$(1,1)$

$\frac{1}{x}=\frac{1}{4}x \quad 4=x^2 \quad x=2$ since x is 2 y is $\frac{1}{2}$

$(2,\frac{1}{2})$

$$A=\int_0^1\left(x-\frac{1}{4}x\right)dx+\int_1^2\left(\frac{1}{x}-\frac{1}{4}x\right)dx$$

$$=\int_0^1\frac{3}{4}x\,dx+\int_1^2\left(\frac{1}{x}-\frac{1}{4}x\right)dx$$

$$\left[\frac{3}{8}x^2\right]_0^1+\left[\ln|x|-\frac{1}{8}x^2\right]_1^2$$

$$\frac{3}{8}+\ln 2-\frac{1}{2}-(0-\frac{1}{8})$$

$$\frac{3}{8}-\frac{4}{8}+\frac{1}{8}=0$$

$\therefore A=\ln 2$