Jack Erwin - Section 7.1

Questions

1. Show that $y = \frac{2}{3}e^{x} + e^{-2x}$ is a solution of the differential equation $y' + 2y = 2e^{x}$

2. Suppose a certain population is modeled by the differential equation

$$\frac{dP}{dt} = 1.2P\left(1 - \frac{P}{4200}\right)$$

A) for what values of P is the population increasing?

B) for what values of P is the population decreasing?

C) What are the equilibrium solutions to this differential equation?

Jack

Solutions

(1.) $y = \frac{2}{3}e^{x} + e^{-2x} \rightarrow$ solution to $y' + 2y = 2e^{x}$

1. take derivitive: $\frac{d}{dx}\left[\frac{2}{3}\cdot e^{x}\right] + \frac{d}{dx}\left[e^{-2x}\right]$

$$\frac{d}{dx}\left[\frac{2}{3}\cdot e^{x}\right] = \frac{2}{3}\frac{d}{dx}\left[e^{x}\right] + \frac{d}{dx}\left[e^{-2x}\right] =$$

$$\frac{2}{3}e^{x} + \frac{d}{dx}\left[e^{-2x}\right] = \frac{2}{3}e^{x} - 2e^{-2x}$$

2. Set equal: $y' + 2y = \left(\frac{4}{3}\right)e^{x} + \cancel{2e^{-2x}} + \left(\frac{2}{3}\right)e^{x} - \cancel{2e^{-2x}} =$

3. Solve: $\boxed{= 2e^{x}}$ so $y = \frac{2}{3}e^{x} + e^{-2x}$ is a solution to $y + 2y = 2e^{x}$ ✓

(2.) A) $\frac{dP}{dt} = 1.2p\left(1 - \frac{P}{4200}\right)$ $\frac{dP}{dt} > 0$

$P > 0$ & $P < 4200$

* for population increasing $\boxed{[P \in (0, 4200)]}$

B) $\frac{dp}{dt} < 0$ so $1.2p\left(1 - \frac{p}{4200}\right) < 0$

$p < 0$ & $p > 4200$

* so for population decreasing $\boxed{[p \in (4200, \infty)]}$

C) $\frac{dp}{dt} = 0$ $1.2p\left(1 - \frac{p}{4200}\right) = 0$

$p = 0$ & $p = 4200$

* equilibrium solutions $\boxed{[p = 0, 4200]}$

JACOB RAMIREZ 7.3 SEPARABLE EQUATIONS

Find the equation of the function cointaining the given point.

① $\frac{dy}{dx} = \frac{2x + \sec^2 x}{2y}$; $y(0) = -5$

② $\frac{dy}{dx} = 3x + 3xy^2$; $y(0) = \sqrt{3}$

JACOB RAMIREZ 7.3 SEPARABLE EQUATIONS

SOLUTIONS

① $\frac{dy}{dx} = \frac{2x + \sec^2 x}{2y}$; $y(0) = -5$

$2y\,dy = (2x + \sec^2 x)\,dx$

$\int 2y\,dy = \int (2x + \sec^2 x)\,dx \Rightarrow \int 2y\,dy = \int 2x\,dx + \int \sec^2 x\,dx$

$y^2 = x^2 + \tan x + C \Rightarrow y = \pm\sqrt{x^2 + \tan x + C}$

plug in $y(0) = -5 \Rightarrow -5 = \pm\sqrt{0^2 + \tan(0) + C}$ $\quad -5 = \pm\sqrt{C}$

$-5 = -\sqrt{C}$; $C = 25 \Rightarrow \boxed{y = -\sqrt{x^2 + \tan x + 25}}$

② $\frac{dy}{dx} = 3x + 3xy^2$; $y(0) = \sqrt{3}$

$\frac{dy}{dx} = 3x(1 + y^2) \Rightarrow \frac{1}{1+y^2}\,dy = 3x\,dx$

$\int \frac{1}{1+y^2}\,dy = \int 3x\,dx \Rightarrow \arctan y = \frac{3x^2}{2} + C$

plug in $y(0) = \sqrt{3} \Rightarrow \arctan\sqrt{3} = \frac{3(0)^2}{2} + C \Rightarrow \arctan\sqrt{3} = C$

$C = \pi/3$; $\arctan y = \frac{3x^2}{2} + \pi/3 \Rightarrow \boxed{y = \tan\left(\frac{3x^2}{2} + \frac{\pi}{3}\right)}$

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MATH - 151 - 02
Section 4.1

1) Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the point.

$$\left(1, \frac{5\pi}{2}\right)$$

2) Identify the curve by finding a Cartesian equation for the curve.

$$r = 3\sin\theta$$

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Section H.1

2) Identify the curve by finding a Cartesian equation for the curve.

$$r = 3\sin\theta$$

Remember

$$x = r\cos(\theta) \qquad ; \qquad r = \sqrt{x^2+y^2}$$

$$y = r\sin(\theta) \qquad \qquad r^2 = x^2+y^2$$

So...

$$\sin\theta = \frac{y}{r}$$

So...

$$r = 3\sin\theta \longrightarrow r = \frac{3y}{r}$$

So...

$$r^2 = 3y = x^2+y^2 - 3y = 0$$
$$-3y$$

So we get: $\boxed{x^2+y^2-3y=0}$

03/09/23
Ashley Rodriguez

Parametric equations

① Eliminate the parameter for the following set of parametric equations, sketch the graph of the parametric curve & give any limits that might exist on x and y.

$x = 4 - 2t$ $\qquad$ $y = 3 + 6t - 4t^2$

Steps:

① Eliminate the parameter from the set of parametric equations.

→ Solving the "x" equation for "t" and plugging that into the "y" eq.

$x = 4 - 2t$ $\qquad$ $y = 3 + 6t - 4t^2$

$x - 4 = -2t$

$t = \frac{1}{2}(4 - x)$

$y = 3 + 6\left[\frac{1}{2}(4-x)\right] - 4\left[\frac{1}{2}(4-x)\right]^2$

$y = -x^2 + 5x - 1$

② Find the x & y intercepts, and vertex.

- x-intercepts: Found by solving $f(x) = 0$

$-x^2 + 5x - 1 = 0$ $\qquad$ • Note: can also find by using quad. formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad$ $a = -1$ $\quad$ $b = 5$ $\quad$ $c = -1$

$= \frac{-5 \pm \sqrt{(5)^2 - 4(-1)(-1)}}{2(-1)} = \frac{5 \pm \sqrt{21}}{2} = ①\ 0.2087, ②\ 4.7913$

- y-intercept: $(0, f(0)) = (0, -1)$ plug in the 0 to the ~~original~~ equat. $-x^2 + 5x - 1$
- Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

• values obtained from the ~~orig.~~ equat.

$= \left(\frac{-5}{2(-1)}, f\left(\frac{5}{2}\right)\right) = \left(\frac{5}{2}, \frac{21}{4}\right)$

③ Find the direction of motion (direction indicating inc. values of parameter)

→ Plug in values of t to get points.

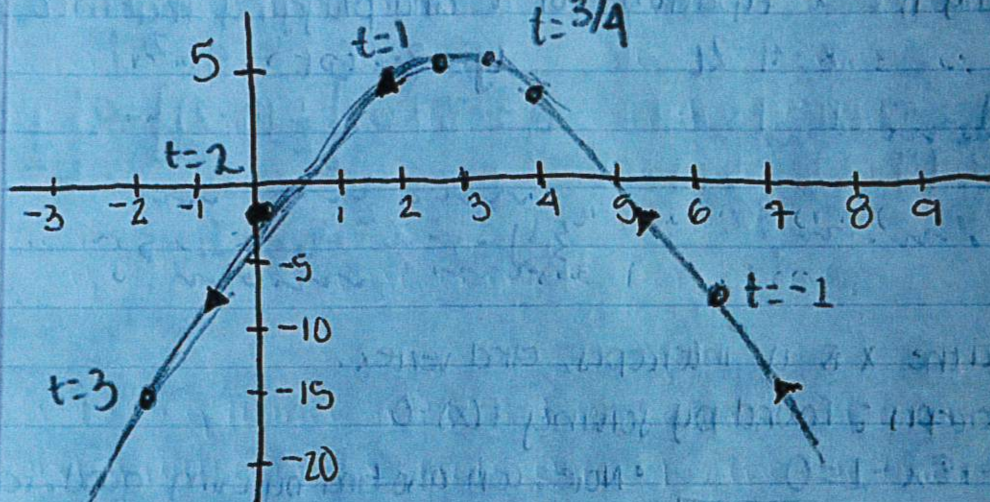
t	x	y
-1	6	-7
0	4	3
3/4	5/2	21/4
2	0	-1
3	-2	-15

④ Limits.

- Because it is a parabola that opens downwards, there's no restriction of any t's. There will not be any limits on x, but for y, it has to be the value from the vertex.

$$-\infty < x < \infty \qquad y \leq \frac{21}{4}$$

⑤ Sketch



Problem (2)

Eliminate the parameter for the following set of parametric equations, sketch the graph of the parametric curve and give any limits that might exist on x and y.

$x = 3\sin(2t)$ $y = -4\cos(2t)$ $0 \le t \le 2\pi$

Steps:

Eliminate the parameter from the parametric equations.

- by using trig identities $\cos^2(\theta) + \sin^2(\theta) = 1$

$\sin(2t) = \frac{x}{3}$ $\cos(2t) = -\frac{y}{4}$

- Plug into the trig identity.

$$\left(-\frac{y}{4}\right)^2 + \left(\frac{x}{3}\right)^2 = 1 \rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1$$

Getting Limits.

- Finding the largest possible range of limits for x & y:

$-1 \le \sin(2t) \le 1$ $-1 \le \cos(2t) \le 1$

$-3 \le 3\sin(2t) \le 3$ $4 \ge -4\cos(2t) \ge -4$

$-3 \le x \le 3$ $-4 \le y \le 4$

- Start with the appropriate trig function and build up the eqtn. for x & y by first multiplying the trig function by any coeff. and then adding/subtracting any numbers.

Direction of motion. (indicates increasing values of the param.)

Instead of using the table, we'll look at how the behavior of sine/cosine increase.

$t = 0$ parametric curve will start $(0, -4)$

$\sin(2t)$ increase from 0 to 1 $\cos(2t)$ decrease from 1 to 0.

$x = 3\sin(2t)$ increase from 0 to 3.

$y = -4\cos(2t)$ increase from -4 to 0.

"cosine is decreasing the minus sign on the coeff. means the "y" will be increasinsing.

- Behavior of x & y must be happening simultaneously.

→ The only possibility is to start the parametric curve at $(0,-4)$ increasing the value of t ~~it~~ must move to the right in counterclockwise until reaching $(3,0)$.

Point $(3,0)$

$\sin(2t)=1$ $\cos(2t)=0$

$\sin(2t)$ dec. from 1 to 0, decrease from 0 to -1.

→ x will now decrease from 3 to 0, while y will continue to increase from 0 to 4

- Another increase in "t" will force x to decrease from 0 to -3 & y will have to also decrease from 4 to 0.

→ Continuing to increase "t", x will increase from -3 to 0 and y will decrease from 0 to -4, the direction of the ellipse is counter clockwise $(-3,0)$ to $(0,-4)$.

→ Determine the values of 2t in order to look for the values of sine and cosine.

$t>0$ $\sin(2t)=0$ at $2t=\pi, 2\pi, 3\pi$

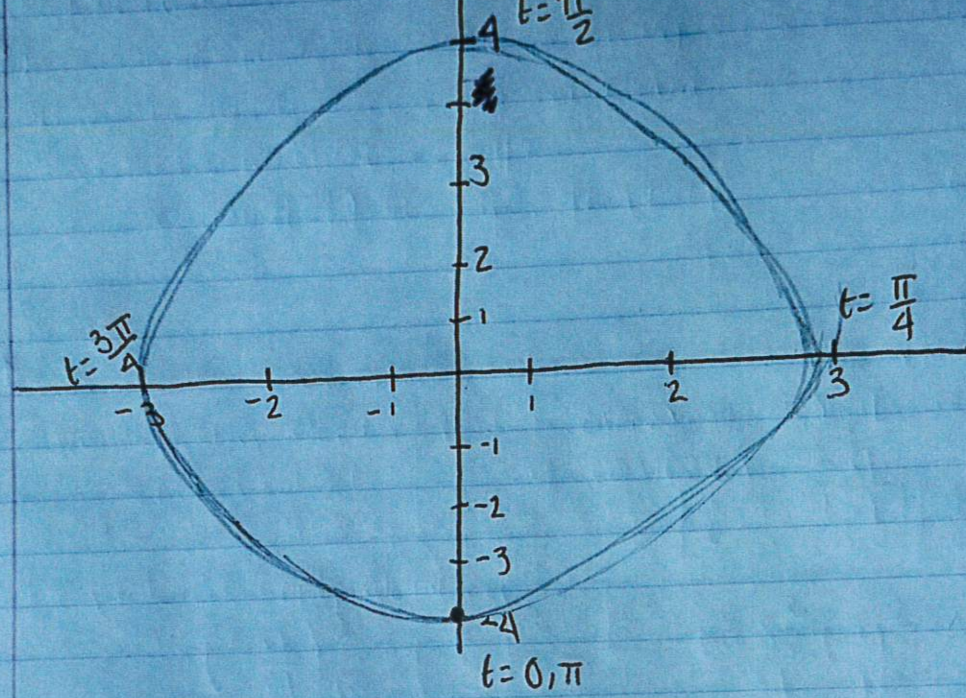
$\cos(2t)=1$ at $2t=2\pi, 4\pi, 6\pi$

- First value of $2t=2\pi$, is in both lists

$2t=2\pi$ $t=\pi$

→ We will get back to the starting point, when we reach $t=\pi$

Ellipse's range $\pi \leq t \leq 2\pi$



t = π/2
t = π/4
t = 3π/4
t = 0, π
4
3
2
1
-1
-2
-3
-4
-2
-1
1
2
3