Math 320 Linear Algebra
Assignment # 12

1. Suppose that $A, B, C \in \mathbb{R}^{n\times n}$ with $A \sim B$ and $B \sim C$. Show that $A \sim C$.

2. Let

$$A = \begin{bmatrix} 14/3 & -5/3 & 1 \\ 17/3 & -8/3 & 1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Consider:

$$\mathscr{B} = \left\{ \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \right\}.$$

   (a) Show that $\mathscr{B}$ is an eigenbasis with respect to A and find the corresponding eignenvalues?

   (b) Find D and P so that $A = PDP^{-1}$.

3. Find an eigenbasis for

$$A = \begin{bmatrix} -3 & -3 & 6 \\ 0 & 0 & -6 \\ 0 & 0 & -3 \end{bmatrix}$$

.

4. Consider the matrix

$$A = \begin{bmatrix} -13 & -10 & 5 \\ 20 & 17 & -10 \\ 10 & 10 & -8 \end{bmatrix}$$

   (a) Find $\text{tr}(A)$ To learn about the trace of a matrix, see the short video: Trace of a Matrix

   (b) Find $\det(A)$ (You can use an online calculator if you don't need more practice).

   (c) Show $\text{char}(A) = -x^3 - 4x^2 + 3x + 18$ (the characteristic polynomial of A).

   (d) Find the eigenvalues of A. (Hint: the eigenvalues are integers so you can find roots by graphing the polynomial)

   (e) Find an eigenbasis for $\mathbb{R}^3$ with respect to A. (You can use an online row-reducing calculator unless you need more practice)

   (f) Find invertible matrix P and diagonal matrix D so that $A = PDP^{-1}$.