Math 320 Linear Algebra
Assignment # 7

1. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and define $\det(A) = ad - bc$.
   (a) Show that if $\det(A) = 0$ then A is does not row reduce to the identity matrix and hence is not invertible (i.e is singular). (Hint use two cases, $a = 0$ and $a \neq 0$.)
   (b) Conversely show that if $\det(A) \neq 0$ then A is invertible and $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ by showing that $AA^{-1} = I_2$.

2. Consider:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & -2 & 5 \\ 0 & 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -2 & 4 \\ 1 & 2 & 3 & 5 & -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 & 5 & -1 & 2 \\ 0 & 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

   where B is the output of doing Gaussian elimination on A.
   (a) Find elementary matrices $E_1, E_2, \ldots, E_r$ such that:
   $$E_r E_{r-1} \ldots E_2 E_1 A = B$$
   (b) Find an invertible matrix C such that $CA = B$.
   (c) Do the matrix multiplication to show that indeed $CA = B$.
   (d) Find $E_1^{-1}, E_2^{-1}, \ldots, E_r^{-1}$ (that is find the inverse of the elementary matrices found above).
   (e) Use the previous part to find C^{-1}.
   (f) Verify by matrix multiplication that $A = C^{-1}B$.