Problems from Assignment 5

1. For this problem you may use the folloing integrals when $a > 0$ and $n \in \mathbb{N}$:

$$\int_0^a (1-\frac{x}{a})^{n-1}\,dx = \frac{\theta}{n}$$
$$\int_0^a x(1-\frac{x}{a})^{n-1}\,dx = \frac{\theta^2}{n(n+1)}$$
$$\int_0^a x^2(1-\frac{x}{a})^{n-1}\,dx = \frac{2\theta^3}{n(n+1)(n+2)}.$$

Suppose $X_1, X_2, \ldots, X_n \overset{\text{iid}}{\sim} \mathscr{U}(0,\theta)$ and let X_{min} be the minimum of the n values.

(a) Find the pdf of X_{min}. (Hint: $F_{X_{\text{min}}}(x) = 1 - \mathrm{P}(X_{\text{min}} > x)$.)

(b) Check that it is indeed a pdf.

(c) Find $\mathrm{E}(X_{\text{min}})$.

(d) Choose c so $\hat{\theta}_1 = cX_{\text{min}}$ is an unbiased estimator.

(e) Find $\mathrm{Var}(X_{\text{min}})$.

(f) Compare thie to $\mathrm{Var}(X_{\text{max}})$ that we did in class, and explain why this makes sense.

(g) Find $\mathrm{Var}(\hat{\theta}_1)$.

(h) How does the efficency compare to $\hat{\theta} = \dfrac{n+1}{n} X_{\text{max}}$.