Problems from Assignment 14

1. Let $V \sim \chi^2_n$. Also let $X_1, X_2, \ldots, X_n \overset{\text{iid}}{\sim} \mathrm{N}(\mu, \sigma^2)$ and $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X}_n)^2$.

   (a) Show:

   $$E\left(\sqrt{V}\right) = \frac{\sqrt{2}\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$$

   Remember $E(g(X)) = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$. And of course, you don't do the integration you relate it to integrals you already know.

   (b) Find $\mathrm{E}(S)$.

   (c) We have shown the S^2 is an unbiased estimator of σ^2. Is S an unbiased estimator of σ?

2. Suppose $X \sim \chi^2_m$ and $Y \sim \chi^2_n$, with X and Y independent. Use moment generating functions (of the chi-squared distribution) to show that $X + Y \sim \chi^2_{m+n}$.