Problems from Assignment 12

1. (a) Suppose X and Y are independent random variables and suppose the $f_Y(y)$ is an even function. Show that $f_{Y/X}(w)$ is an even function.

   (b) Show if $T \sim t_n$ then f_T is an even function. You may not use the density of T since we computed that using the fact that f_T is symmetric around the origin which you are now proving.

2. Let $T \sim t_5$.

   (a) Write $f_T(t)$ without any Γ in it.

   (b) Use Simpson's rule with $n = 6$ intervals to estimate $\mathrm{P}(0 \leq T \leq 1.5)$.

   (c) Using the above estimation, estimate $P(T \geq 1.5)$.

   (d) Is this consistent with Table A.2 in the book?

3. Suppose $X, Y \overset{\text{iid}}{\sim} N(0,1)$ and let $W = \dfrac{Y^2}{X^2}$.

   (a) Find $f_W(w)$ (it should not include any Γ functions).

   (b) Argue what $\mathrm{P}(W \leq 1) = \dfrac{1}{2}$ without doing integration.

   (c) By doing a clever substitution, integrate $f_W(w)$ to show that $\mathrm{P}(W \leq 1) = \dfrac{1}{2}$.