Combinatorics Homework

Homework Due on January 29, 2015

1. A California license plate has a number followed by three letters followed by three numbers.
   (a) How many total license plates are possible?
   (b) Without doing any calculation do you think there are more license plates in which all the numbers and letters are different or ones that don't have this property?
   (c) How many license plates have all different numbers and letters?
   (d) How many license plates have at least one number or letter appear more than once?
2. For each n determine how many functions there are between $\{1, 2, 3, \ldots, n\}$ and $\{1, 2, 3, \ldots, n\}$ that are not one-to-one.
3. A company has 18 employees, seven males and eleven females. How many committees of 5 employees can be formed that do not consist of entirely one gender?
4. You have eight friends attending a ball game and you bought eight tickets together in one row. Two of the friends formerly dated each other. How many ways are there to sit the friends in those seats so the two exes are not seated next to each other?
5. You will prove that for all $1 \leq k \leq n$:

$$n\binom{n-1}{k-1} = k\binom{n}{k}.$$

   In two ways:
   (a) Using a combinatorial argument.
   (b) Using an analytic argument