

Combinatorics Homework

Homework Due on February 26, 2015

Do the following problems and, of course, give explanation when needed.

- Find a sequence of length 6 that has no increasing subsequences of length 3 and no decreasing sequences of length 4.
 - Prove that every sequence of length 7 either has an increasing subsequence of length 3 or a decreasing subsequence of length 4.
- Let G be a connected graph. Prove G has an Eulerian path from vertex s to t if and only if the degree of s and t are odd and the degree of all other vertices are even.
- Show any graph has an even number of vertices of odd degree.
- If G is a graph with n vertices, what is the most edges it could have (remember all of our graphs are simple).
- Draw (with explanation) a connected graph with at least 6 vertices that:
 - Has a closed Eulerian path but no Hamiltonian cycle.
 - Has a Hamiltonian cycle but no closed Eulerian path.
 - Has no closed paths (such a graph is called a tree).
- A simple graph is called regular if all its vertices have the same degree. Let G be a regular connected graph with 22 edges. What are all the possible values for the number of vertices it can have?
- The following graph is know as the Petersen graph:

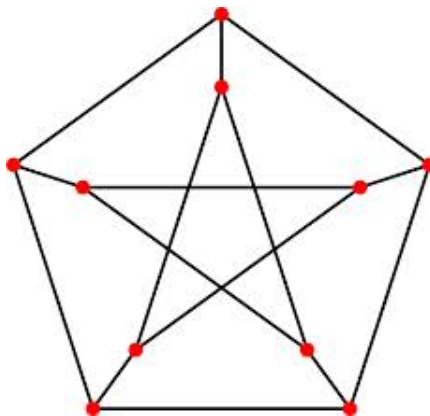


Figure 1: Petersen Graph

- Is it regular?
 - Does it contain an Eulerian path?
 - Does it contain a Hamiltonian cycle?
- Is there a graph where the vertices have degrees: 4,4,4,2,1,1?