Combinatorics Homework
Homework Due on February 5, 2015

1. How many different permutations of $\{1, 2, 3, 4, 5, 6\}$ have a 2 in the second place?

2. How many odd numbers between 1000 and 9999 have distinct digits?

3. Consider the identity:
$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}.$$

   Prove it

   (a) using a combinatorial argument.

   (b) using an analytic argument

4. A poker hand is called "two pairs" if contains two different pairs but is not a full house. Calculate the probability of a random hand being two pairs.

5. In a certain lottery they have 8 machines each with balls labeled 1 to 9. A ball comes out of each machine and then the balls are put in order (so one outcome would be 11567999). How many outcomes are there?

6. An NHL team plays 82 in a regular season, how many final records are possible if:

   (a) A team can either win, lose, or lose in overtime(which is counted differently) in each game?

   (b) If only wins and loses are counted?

7. You win a contest where you get to choose a total of 20 free tickets to any Charges games. There are 8 home games in each season, how many ways are there to do that? (One possibility is you choose 3 to game 1, 7 to game 7, and the remaining 10 to game 8.)

8. Prove the following identity:
$$\binom{k}{k} + \binom{k+1}{k} + \binom{k+2}{k} + \ldots + \binom{n}{k} = \binom{n+1}{k+1}.$$