Combinatorics Homework

Homework Due on April 16, 2015

Do the following problems and, of course, give explanation when needed.

1. A four-sided die is shaped like a tetrahedron an the numbers $1-4$ can be rolled with equal probability. Suppose that two such die are rolled and the results are summed:

   (a) Write down all the possible sums and how many ways there are to make each one.

   (b) Use generating functions to find another way to label the dice with positive integers to get the same possibilities with the same number of ways to get each.

2. Find an explicit formula for a_n if $a_0 = 0$ and $a_{n+1} = a_n + 2^n$ for $n \geq 0$.

3. A child is climbing a staircase and can either go up one or two steps at a time. Let a_n be the number of ways the child can get to the nth step. Find an explicit formula for a_n.

4. This problem is a similar setup to the one we did in class. Suppose that I have n students in my class and my roster is ordered in alphabetical order. I will choose some point on my list and have the first part of the list work on project A and the second part work on project B. However, I will now choose some arbitrary number of students from each group to give the presentation. (Note: there may be 0 to n people in each group and the number of presenters can be 0 to everyone in the group.)

   (a) Use generating functions to find the number of ways to do this for each n.

   (b) Give a combinatorial proof of the above result.