

# Combinatorics Homework

Homework Due on April 30, 2015

Do the following problems and, of course, give explanation when needed.

1. Find a simple, closed form for the an ordinary generating function of the sequence  $a_n = n^2$ .
2. (a) Prove that the number of partitions of  $n$  into exactly  $k$  parts is equal to the number of partitions of  $n$  in which the largest part is exactly  $k$ . (Hint: Remember those Young's Diagrams I showed in class, use those to give a bijection.)  
(b) Prove that the number of partitions of  $n$  into at most  $k$  parts is equal to the number of partitions of  $n + k$  into exactly  $k$  parts.  
(c) Let  $p_k(n)$  be the number of partitions of  $n$  into exactly  $k$  parts. Find the ordinary generating function of the sequence  $p_k(n)$  for a fixed  $k$ .