

Week of October 7, 2019:

Problem: Arrange positive integers 1,2,...,15 into a 3x5 array with equal row sums and equal column sums.

Hint: What are the row and column sums? Work on equal column sums first. There aren't many ways to position 11,12,...,15 into columns with the column sum restriction. For row sums, keep in mind there must be an even number of odd integers in each row.

Solution:

1	2	12	14	11
10	15	3	4	8
13	7	9	6	5

Week of October 14, 2019:

Problem: What is the probability that n randomly chosen points on the circumference of a circle all lie in the same semicircle.

Hint: A point is a leader if it begins a semicircle in the clockwise direction which includes all other points. Note that a leader is unique if it exists. What is the probability that a given point is a leader?

Solution: Following the hint, each of the $n-1$ remaining points has a probability of $\frac{1}{2}$ of being in the semicircle, so the probability that they are all in this semicircle is $(\frac{1}{2})^{n-1}$. Since each of the n points is equally likely to be the unique leader, the probability that all n lie in the same semicircle is $n/(2^{n-1})$.

Week of October 21, 2019:

Problem: What is the sequence below, and what are its next elements? (The numbers in parentheses provide a helpful hint.)

1 (1), 2 (2), 4 (3), 6 (4), 12 (6), 24 (8), 36 (9), 48 (10), 60 (12), 120 (16), 180 (18),
240 (20), 360 (24), 720 (30), 840 (32), ...

Hint: divisors

Solution: The next natural number with more divisors than the previous one, so the next number after 840 would be 1260 (with 36 divisors). (Puzzle by Guy Moore.)

Week of October 28, 2019:

Problem: Show that any odd integer n not a multiple of 5 divides a number consisting of all 1s. For instance, 3 divides 111 and 7 divides 111,111.

Hint: Pigeonhole principle. Also, what can be said about n if it divides a number consisting of 1s followed by 0s?

Solution: If n doesn't evenly divide any number consisting of all 1s then, since there are only $n-1$ remainders other than 0, there must be two such numbers with the same remainder, by the Pigeonhole principle, and the difference of those numbers (a number consisting of 1's followed by 0s) is then divisible by n . But since n has no factors of 2 or 5, it cannot divide any power of 10 and so must divide the number consisting of the leading 1s.