Taylor Series Fun Pack !

Here's some more practice problems for some of the harder Taylor Series Applications problems. This is for extra practice for these specific problems only – it does not cover everything that will be on the final exam! You should look at the assigned homework problems to get a good idea of the range of problems that you can expect on the exam. Have fun!

1. Use series to approximate the values of the following integrals with an error of less than 10^{-4}

   (a) $\displaystyle\int_0^{1/2} e^{-x^3}\,dx$ (0.4849)

   (b) $\displaystyle\int_0^{1} x\sin(x^3)\,dx$ (0.1848)

   (c) $\displaystyle\int_0^{1/2} \frac{\arctan x}{x}\,dx$ (0.4872)

   (d) $\displaystyle\int_0^{1/64} \frac{\arctan x}{\sqrt{x}}\,dx$ (.0013)

2. Evaluate the following limits:

   (a) $\displaystyle\lim_{x\to 0} \frac{\sin x - \arctan x}{x^3}$ (1/6)

   (b) $\displaystyle\lim_{x\to 0} \frac{3x - \arctan(3x)}{4x^3}$ (9/4)

   (c) $\displaystyle\lim_{x\to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{14x^4}$ $\left(\dfrac{1}{14\cdot 24}\right)$

   (d) $\displaystyle\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$ (1/2)

3. The approximation $\dfrac{1}{e^x} \approx 1 - x + \dfrac{x^2}{2}$ is used when x is small. For what values of x will this degree 2 polynomial give an approximation for $\dfrac{1}{e^x}$ accurate to within 0.001? ([-.1716, .1716])

4. If $\cos x$ is approximated by $1 - \dfrac{x^2}{2}$, then what is the largest that the error will be using this degree 2 polynomial to approximate $\cos x$ for values of x with $|x| < \frac{1}{2}$? $(\frac{1}{48})$

5. (a) Write the MacLaurin Series for $f(x) = \ln(1+x)$.

   (b) Use part (a) to find an exact value for the sum of the alternating harmonic series. [Hint: evaluating at an appropriate x value will give the alternating harmonic series] $(\ln 2)$
   (Thanks for Gabriel M for pointing out this cool fact!)