Laboratory #1: Transmission Line Characteristics

I. OBJECTIVES

Coaxial and twisted pair cables are analyzed. The results of the analyses are experimentally verified using a network analyzer. S_{11} and S_{21} are found in addition to the characteristic impedance of the transmission lines.

II. INTRODUCTION

Two commonly encountered transmission lines are the coaxial and twisted pair cables. Coaxial cables are found in broadcast, cable TV, instrumentation, high-speed computer network, and radar applications, among others. Twisted pair cables are commonly found in telephone, computer interconnect, and other low speed (<10 MHz) applications. There is some discussion on using twisted pair cable for higher bit rate computer networking applications (>10 MHz).

The characteristic impedance of a coaxial cable is,

$$Z_o = \sqrt{\frac{L}{C}} = \frac{1}{2p}\sqrt{\frac{m}{e}}\ln\left(\frac{b}{a}\right), \tag{1}$$

so that

$$Z_o\sqrt{\frac{e_r}{m_r}} = 60\ln\left(\frac{b}{a}\right) = 138\log\left(\frac{b}{a}\right). \tag{2}$$

The dimensions *a* and *b* of the coaxial cable are shown in Figure 1.

L is the line inductance of a coaxial cable is,

$$L = \frac{m}{2p}\ln\left(\frac{b}{a}\right)\text{[H/m]} \quad . \tag{3}$$

The capacitor per unit length of a coaxial cable is,

$$C = \frac{2pe}{\ln\left(b/a\right)}\text{[F/m]} \quad . \tag{4}$$

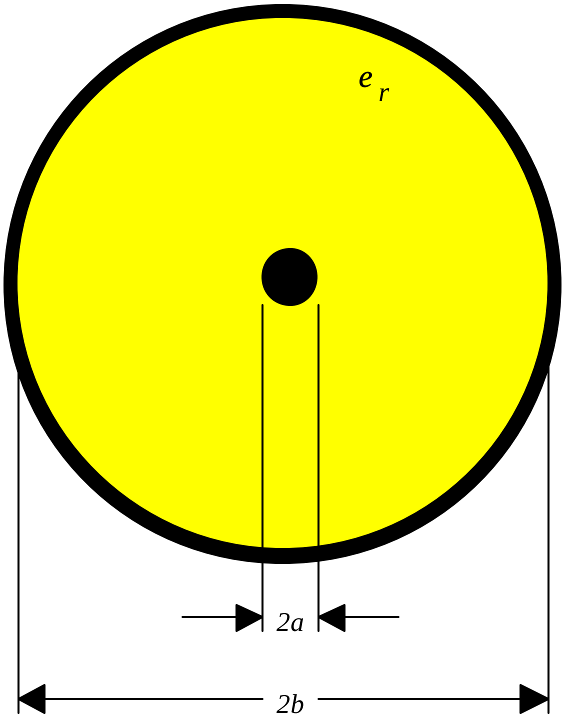

Figure 1. Coaxial Cable Dimensions

The two commonly used coaxial cables are the RG-58/U and RG-59 cables. RG-59/U cables are used in cable TV applications. RG-59/U cables are commonly used as general purpose coaxial cables.

The RG-58/U coaxial cable have typical dimensions of:

Inner conductor diameter: 0.032 in.
Polyethylene core diameter: 0.116 in.
Polyethylene e_r : 2.3
Outer conductor diameter: 0.195 in.

The RG-59/U coaxial cable have typical dimensions of:

Inner conductor diameter: 0.023 in.
Polyethylene core diameter: 0.146 in.
Polyethylene e_r : 2.3
Outer conductor diameter: 0.247 in.

The characteristic impedance of a twisted pair cable is,

$$Z_o = \sqrt{\frac{L}{C}} = \frac{1}{p}\sqrt{\frac{m}{e}}\cosh^{-1}\left(\frac{D}{d}\right), \tag{5}$$

and for $D/d >> 1$,

$$Z_o \approx \frac{1}{p}\sqrt{\frac{m}{e}}\ln\left(\frac{2D}{d}\right). \tag{6}$$

The dimensions D and d of the twisted pair cable are shown in Figure 2.

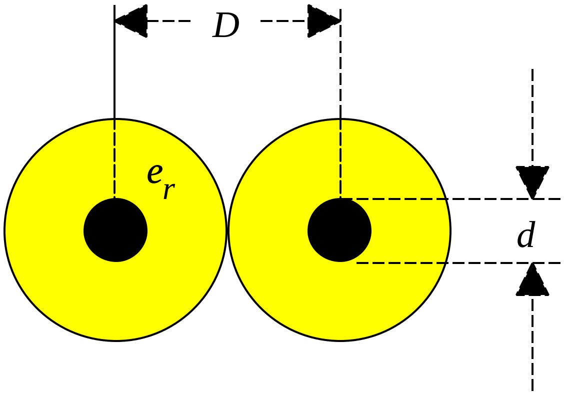

Figure 2. Twisted Pair Cable Dimensions

L is the line inductance of a twisted pair cable is,

$$L = \frac{m}{p}\cosh^{-1}\left(\frac{D}{d}\right)\text{[H/m]} \ . \tag{7}$$

The capacitor per unit length of a twisted pair cable is,

$$C = \frac{pe}{\cosh^{-1}\left(D/d\right)}\text{[F/m]} \ . \tag{8}$$

The relative dielectric constant of the cable jacket is $e_r = 2.5$

One common method for measuring the reflection and transmission characteristics of any device under test (in this case a microstripline) involves the using a network analyzer. A network analyzer allows convenient measurements of signal reflection and transmission in a variety of formats. It can measure signal delay, phase, and gain of the device under test (DUT). All of these measurements are made with respect to the source and terminal impedance of the network analyzer. The default impedance of the HP8752A network analyzer is set at 50 Ω.

The signal reflected from the DUT is usually measured as a ratio to the incident signal. It can be expressed as reflection coefficient or return loss. These measurements are described mathematically as,

$$\text{Reflection coefficient} \equiv \frac{\text{reflected power}}{\text{incident power}} = \left|\frac{E_{refl}}{E_{inc}}\right| = r \quad \text{(magnitude only)}$$
$$= \Gamma \quad \text{(Reflection magnitude and phase)} \tag{9}$$

$$\text{Return loss (dB)} = -20 \log r \tag{10}$$

$$\text{Standing Wave Ratio} \equiv SWR = \frac{1+r}{1-r} \quad \text{(pronounced "}\textit{swir}\text{" as in }\textit{swirl}\text{)} \tag{11}$$

Displaying the reflection measurement in polar form on the network analyzer with a marker allows direct determination of the complex impedance of the DUT. The center of the circle represents a coefficient Γ of 0 (a perfect match, no reflected signal). The outermost circumference of the scale represents a Γ of 1 (100% reflection). The phase angle is directly read from the display. The magnitude and phase will be directly displayed in the marker data readout for any frequency.

The amount of power reflected from a device is directly related to the impedances of the DUT and the measurement instrument. $\Gamma = 0$ occurs when the DUT and the analyzer have identical impedances. A short circuit has $\Gamma = 1 \angle 180°$. Every other value of Γ corresponds uniquely to a complex device impedance. In terms of impedances,

$$\Gamma = \frac{Z_{DUT} - Z_o}{Z_{DUT} + Z_o}, \tag{12}$$

where Z_o is the impedance of the measurement instrument,
Z_{DUT} is the impedance of the DUT.

To facilitate computations, the normalized (in this case normalized to 50 Ω) impedance is,

$$Z_N = \frac{Z_{DUT}}{Z_o} = \frac{1+\Gamma}{1-\Gamma}. \tag{13}$$

S-parameters are commonly used to characterize high frequency circuits. S-parameters (or Scattering-parameters) basically are two-port characteristics of the DUT. Additionally, insight into the behavior of traveling waves are readily deduced from S-parameters.

S-parameters can readily be found using the schematic of the test set-up shown in Figure 3.

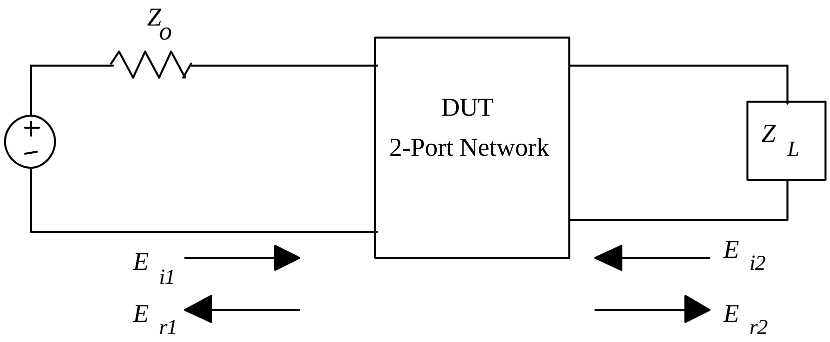

Figure 3. Two Port Network Used For S-Parameter Measurements

Define new variables with respect the a characteristic impedance of the measurement instrument,

$$a_1 = \frac{E_{i1}}{\sqrt{Z_o}}, \qquad a_2 = \frac{E_{i2}}{\sqrt{Z_o}},$$
$$b_1 = \frac{E_{r1}}{\sqrt{Z_o}}, \qquad b_2 = \frac{E_{r2}}{\sqrt{Z_o}}. \tag{14}$$

S-parameters relates these four waves as follows:

$$\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}. \tag{15}$$

For S_{11}, the output port of the DUT is terminated (with Z_o = 50 Ω) and the ratio of b_1 to a_1 is measured,

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}. \tag{16}$$

Terminating the output port of the DUT with the impedance of the measurement instrument is equivalent to setting a_2 = 0 since a traveling wave incident on this load will be totally absorbed. S_{11} is the input reflection coefficient of the DUT.

The forward transmission through the DUT is the ratio of b_2 to a_1. This could either be the gain of the amplifier or the attenuation of a passive network,

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}. \tag{17}$$

By terminating the input side of the network, we set a_1 = 0 and can then measure the output reflection coefficient, S_{22} , and the reverse transmission coefficient, S_{12} , defined as,

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}, \qquad (18)$$

and

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}. \qquad (19)$$

S-parameters are typically expressed as a magnitude and phase.

III. PROCEDURE

A. Calculate the characteristic impedance of the RG-58/U and RG-59/U coaxial cables

Use both MathCAD and AppCAD to determine the characteristic impedance of the coaxial cables.

B. Confirm the characteristic impedance of the RG-58/U and RG-59/U coaxial cables

Plot the magnitudes of the transmission coefficient, reflection coefficient, and *SWR* of the circuits over a frequency range of 300 kHz to 900 MHz.. Find the S_{11} and S_{21} at 10.7 MHz, 49 MHz, 900 MHz. Determine the impedance of the cables at those frequencies.

C. Calculate the characteristic impedance of the twisted pair cable

Use MathCAD to determine the characteristic impedance of the coaxial cables.

D. Confirm the characteristic impedance of the twisted pair cable

Plot the magnitudes of the transmission coefficient, reflection coefficient, and *SWR* of the circuits over a frequency range of 300 kHz to 10 MHz.. Find the S_{11} and S_{21} at 1.5 MHz and 10 MHz. Determine the impedance of the cable at those frequencies.

E. Comment On Your Results