$$\frac{}{\vdash (\,|\,) : ()} \qquad \frac{\begin{array}{c} \Theta \subseteq \Gamma \qquad \Gamma \setminus \Theta \vdash (\vec{M}, \dots, \vec{N}, \vec{P}, \dots, \vec{Q} \mid) : (\vec{A}, \dots, \vec{B}, \vec{C}, \dots, \vec{D}) \\ f \in \mathcal{G}(\vec{A}, \vec{F}_{\geq 1}) \qquad \cdots \qquad g \in \mathcal{G}(\vec{B}, \vec{G}_{\geq 1}) \\ h \in \mathcal{G}(\vec{C}, ()) \qquad \cdots \qquad k \in \mathcal{G}(\vec{D}, ()) \\ \sigma : (\vec{F}, \dots, \vec{G}, \Theta) \xrightarrow{\sim} \Delta \\ \sigma \text{ preserves the relative order of } F_1, \dots, G_1 \\ \sigma \text{ preserves the relative order of } \Theta \end{array}}{\Gamma \vdash \Big(\sigma\big(\vec{f}(\vec{M}), \dots, \vec{g}(\vec{N}), \Theta\big) \mid h(\vec{P}), \dots k(\vec{Q})\Big) : \Delta}$$

Note that no judgment with more than zero scalar terms can appear as a premise.

Type-checking algorithm and proof of unique derivations. Suppose given a putative judgment $\Gamma \vdash (\vec{R} \mid \vec{Z}) : \Delta$. Since scalars are never variables, we have $\vec{Z} = (h(\vec{P}), \dots k(\vec{Q}))$ for uniquely determined $h, \dots, k$ and $\vec{P}, \dots, \vec{Q}$. We must take Θ to be the list of all variables occurring as terms in $\vec{R}$, in the same order, and $f, \dots g$ to be the list of all generators appearing as the final applications on all other terms in $\vec{R}$, in the order determined by their first components. Now the permutation σ with the given properties is uniquely determined (if it exists), and there are no more choices to make.